

Generalized design framework for collective defined contribution pension plans and benefit smoothing mechanisms

by

Vatsal Desai

B.Sc., St. Xavier's College, 2021

Project Submitted in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

in the
Department of Statistics and Actuarial Science
Faculty of Science

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SIMON FRASER UNIVERSITY
Summer 2025

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Declaration of committee

Name: Vatsal Desai

Degree: Master of Science

Thesis title: Generalized design framework for collective defined contribution pension plans and benefit smoothing mechanisms

Committee:

Chair: Brad McNeney
Associate Professor, Statistics and Actuarial Science

Barbara Sanders
Co-Supervisor
Associate Professor, Statistics and Actuarial Science

Jean-François Bégin
Co-Supervisor
Associate Professor, Statistics and Actuarial Science

Yi Lu
Committee Member
Professor, Statistics and Actuarial Science

Catherine Donnelly
Examiner
Professor, Actuarial Mathematics and Statistics
Heriot-Watt University

Abstract

Existing literature on collective defined contribution (CDC) pensions typically contrasts two approaches to intergenerational risk sharing: full and immediate experience recognition, prioritizing benefit security (e.g., variable payment life annuities and group self-annuitization plans), and deferral of experience to maximize smoothing (e.g., the Royal Mail CDC plan). This report introduces a generalized design framework encompassing both CDC arrangements with ongoing contributions and decumulation-only structures for retirees entering with preexisting assets. By incorporating two key parameters—the immediate experience recognition rate and the smoothing window length—the proposed framework allows plan designs to exist along a spectrum between these extremes. A comprehensive set of performance metrics is used to assess benefit security, stability, and intergenerational equity. We demonstrate how adjusting the immediate experience recognition rate and the smoothing window length enables stakeholders to explore the trade-offs between the objectives and fine-tune plan designs to align more closely with regulatory goals and welfare considerations.

Keywords: collective defined contribution; pension plan design; benefit smoothing; intergenerational risk sharing; membership welfare

Acknowledgements

This thesis represents the culmination of a journey marked by intellectual challenge, growth, and constant learning. I owe immense gratitude to my supervisors, Prof. Barbara Sanders and Dr. Jean-François Bégin, for their exceptional mentorship and for introducing me to the world of pension research. Their thoughtful guidance, academic rigour, and unwavering support—across research, funding, and life—formed the foundation upon which this work was built. I am also thankful to Dr. Yi Lu, my committee member, for her insightful feedback and for the lessons in discipline and time management I gleaned while TAing for her. My sincere thanks extend as well to Dr. Catherine Donnelly, who generously agreed to serve as my examiner across time zones. I must also acknowledge Dr. Richard Lockhart, whose brilliance and kindness left a lasting impression on me through the courses I took with him and the TAship that followed. His influence, though not directly tied to this thesis, profoundly shaped my academic journey.

To my peers in the RiskDynamics group—Louis, Golara, Nikhil, Alice, Jingyu, Isaac, Sara, and Parnian—thank you for the conversations, presentations, and insights. Beyond research, your humour and company made this experience far richer. I’m equally thankful to Tianxing, Hashan, Yirong, Bingfan, Diksha, Nadia, and Hao Wei—for the assignment marathons, laughter-filled nights, and endless mutual venting that made the workdays survivable. A heartfelt shout-out goes to Yingfei Sun, who, by sheer chronological advantage and shared research theme, became my unofficial senior and go-to person for nearly everything. From formatting finicky L^AT_EX tables to issues with citations and even tricky conversations with JF and Barbara, she had always done the same just a week before me, and knew what not to do.

Finally, my deepest gratitude goes to Aman, Kshitij, and Piyush—my housemates, friends, and everyday companions—thank you for your warmth, humour, and presence, which carried me through more than you know. To Devangi and Nihar, thank you for being the people I could always count on to listen, vent, and recalibrate. And most of all, to my parents, Krupa and Dhaval Desai—this thesis is for you. Your sacrifices, your strength, and your love made everything possible. No words can capture what you’ve done for me, but I hope this milestone offers a glimpse of what your belief in me has made real.

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Chapter 1

Introduction

The gradual decline of traditional defined benefit (DB) pension plans is a well-documented phenomenon over recent decades. As reported by the Organisation for Economic Co-operation and Development (2024; OECD hereafter), DB plans constituted less than 40% of total pension assets in most reporting jurisdictions by 2023. A significant portion of the assets currently held in DB plans reflect legacy arrangements—plans that accumulated contributions and reserves during earlier periods with more favourable demographic and economic conditions. Thus, the continued presence of DB assets in the global pension landscape is largely attributable to these older, prefunded schemes rather than ongoing accruals under new DB plans.

This shift—driven by demographic pressures and labour-market changes, particularly within the private sector—has established defined contribution (DC) plans as the predominant retirement savings vehicle across numerous industrialized economies. While DC plans relieve sponsors of funding risk and facilitate individual asset accumulation, they transfer significant investment and longevity risks to members, many of whom lack sufficient financial literacy or behavioural discipline to effectively manage these uncertainties. Notably, DC outcomes demonstrate considerable sensitivity to equity-market fluctuations, and the absence of structured decumulation strategies leaves retirees dependent on heuristic withdrawal approaches (e.g., the 4% rule), which often produce excessive volatility in retirement income.

In response to these limitations within conventional DC arrangements, collective defined contribution (CDC) plans and target benefit plans (TBPs) have emerged as effective hybrids of DB and DC designs, offering enhanced predictability while maintaining adaptability. As illustrated in Figure 1.1, the proportion of new pension assets allocated to such hybrid plans across OECD nations has shown consistent growth from 2007 to 2022, reflecting increased recognition of their potential to reconcile benefit security and stability with sustainable financing.

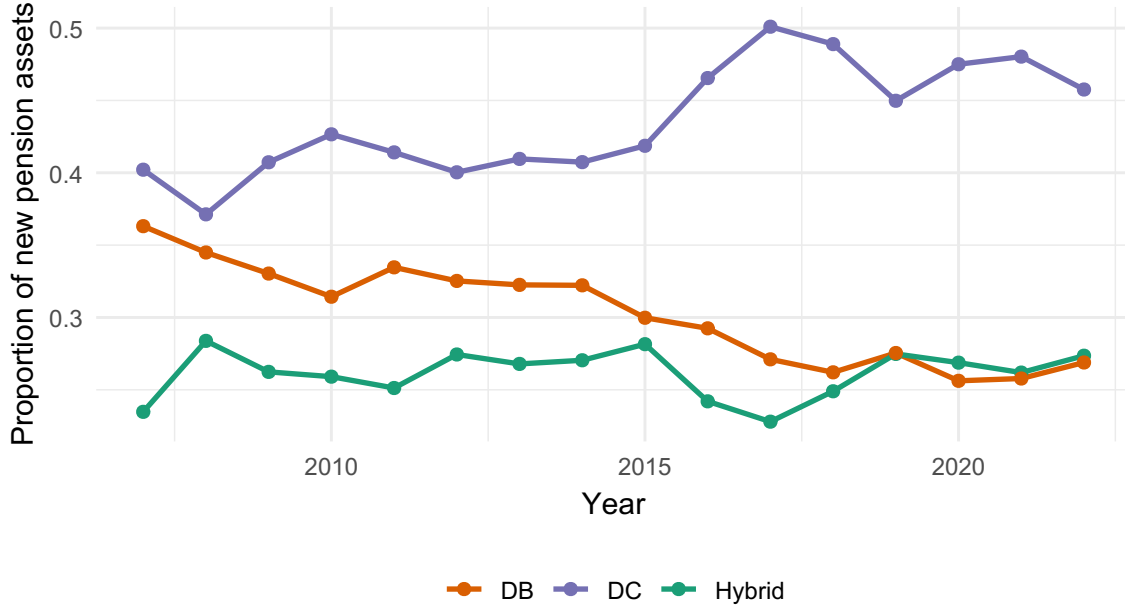


Figure 1.1: Growth of hybrid pension plans in OECD countries.

Notes: This figure illustrates the growth of hybrid pension plans in OECD countries from 2007 to 2022. Here, hybrid is defined as the sum of mixed DB and protected DC. The data can be found in the OECD Data Explorer.

These CDC plans adjust benefits according to realized mortality and investment experience (Wang et al., 2018). Jurisdictions implementing similar shared-risk and TBP frameworks include Canada (e.g., Munnell and Sass, 2013), the United Kingdom (e.g., Department for Work and Pensions, 2014; Thurley, 2014; Armstrong et al., 2024), and the Netherlands (e.g., Kortleve, 2013; Bovenberg et al., 2016; Balter et al., 2020). Across these models, indexation (if any) is conditional, and accrued entitlements may be reduced if funding deteriorates through unfavourable economic or demographic experience.

The implementation of risk-sharing plans can take two distinct forms: decumulation-only models focused solely on the retirement phase, or frameworks accommodating both active contributing members and retirees. This distinction often derives from strategic considerations related to the sponsoring plan’s existing structure. A maturing DC plan might implement a decumulation-only CDC structure to mitigate longevity and benefit-volatility risks without modifying its accumulation phase, whereas a DB plan pursuing sustainability might adopt an integrated CDC design encompassing both accrual and decumulation components. Consequently, a robust, flexible CDC framework must support both configurations, enabling stakeholders to align plan design with demographic composition, governance objectives, and long-term policy goals.

The academic literature has examined two primary approaches to designing these risk-sharing plans. The first approach emphasizes benefit security through various mortality-

pooling mechanisms inspired by historical tontine designs. Notable examples for decumulation-only plan designs include group self-annuitization schemes (Piggott et al., 2005; Valdez et al., 2006; Qiao and Sherris, 2013; Hanewald et al., 2013), pooled annuity funds (Stamos, 2008; Donnelly et al., 2013), annuity overlay funds (Donnelly et al., 2014; Donnelly, 2015), and modern retirement tontines (Milevsky and Salisbury, 2015, 2016; Chen and Rach, 2019; Fullmer, 2019; Iwry et al., 2020; Gemmo et al., 2020; Weinert and Gründl, 2021; Chen et al., 2021). Variable annuities (Balter et al., 2020; Dees et al., 2021) and variable payout life annuities (VPLAs) (Horneff et al., 2010; Boyle et al., 2015) extend this approach by incorporating real-time investment and mortality experience into benefit adjustments. Although these decumulation-only designs enhance income security through immediate experience recognition, they frequently introduce substantial benefit volatility—an undesirable characteristic for retirees seeking predictable income.

The second approach prioritizes benefit stability through extensive smoothing and deferred experience recognition, effectively distributing risk across generations over extended time horizons. Decumulation-only applications of smoothing methods have been investigated by Guillen et al. (2006), Maurer et al. (2016), Balter and Werker (2020), and Kabuche et al. (2025). A prominent example incorporating active members is the Royal Mail Group’s CDC plan in the United Kingdom (Armstrong et al., 2024; Mirza-Davies, 2024), which has stimulated significant policy discussion regarding the implications of deferring experience over a very long horizon.

Consequently, the collective pension landscape is currently characterized by two polar archetypes: plans that maximize benefit security through immediate experience recognition, and plans that maximize stability via long-term smoothing. Yet, few existing frameworks permit stakeholders to calibrate the balance between these extremes. To address this gap, the principal objective of this report is to develop a unified, generalized CDC design framework that facilitates flexible adjustment between immediate experience recognition and smoothing duration.

The unified design framework encompasses both CDC plans with ongoing active contributions and decumulation-only arrangements for retirees. The framework is governed by two key design parameters: the immediate experience recognition rate and the smoothing window length through which remaining experience is absorbed. Within this structure, we examine three benefit adjustment mechanisms: (i) full and immediate recognition of experience; (ii) gradual recognition of experience over a fixed smoothing window length; and (iii) a hybrid adjustment mechanism that combines immediate recognition with subsequent smoothing. By varying the immediate experience recognition rate and the smoothing window length, the framework generates a spectrum of plan designs, enabling stakeholders to

calibrate trade-offs among benefit security, benefit stability, and intergenerational equity according to their specific context.

These smoothing and hybrid mechanisms can introduce complex membership welfare trade-offs. Cui et al. (2011) demonstrated that intergenerational risk sharing can yield *ex-ante* welfare improvements relative to optimal individual life-cycle strategies—despite being zero-sum in market-value terms—without systematically disadvantaging any cohort when properly calibrated. Conversely, Bovenberg et al. (2017) cautioned that excessive smoothing, particularly over horizons exceeding members’ expected remaining lifetimes, can create unintended intergenerational transfers and undermine *ex-ante* fairness, which requires that an individual’s lifetime contributions approximate their expected retirement benefits.

Accordingly, this report introduces a comprehensive suite of performance metrics—spanning benefit security, stability, and intergenerational fairness—that are both theoretically sound and readily interpretable by regulators, trustees, and participants. These measures form a multidimensional evaluation framework that assesses financial viability alongside cohort-level distributional impacts.

The results of extensive stochastic simulations illustrate the inherent trade-offs in CDC plan design: immediate experience recognition enhances security yet amplifies short-term volatility, whereas extensive smoothing improves stability but potentially creates persistent intergenerational transfers. The flexible framework developed herein enables explicit calibration of these competing objectives, offering a spectrum of customized solutions rather than a one-size-fits-all approach. By adjusting the immediate experience recognition rate and smoothing window, sponsors and trustees can align CDC arrangements with membership risk preferences, demographic profiles, and regulatory environments, thereby fostering more resilient and equitable retirement-income systems.

This report is organized as follows. Chapter 2 describes the assumed data-generating processes for investment returns and mortality. Chapter 3 presents the stylized CDC plan frameworks and benefit-adjustment mechanisms. Chapter 4 details the simulation methodology and reports the results under varying design parameters. Chapter 5 introduces and applies the performance metrics to evaluate plan outcomes and trade-offs. Finally, Chapter 6 concludes with a summary of findings and suggestions for future research.

Chapter 2

The assumed data generating process

This report employs simplified models to generate future outcomes that approximate real-world scenarios, thereby capturing the underlying mechanisms of data generation. The methodological framework incorporates a financial market model alongside a mortality model, enabling the simulation of benefit evolution dynamics in the context of pension plan design. These models are designed to provide insights into the effects of both demographic and economic factors on pension benefit evolution, offering a robust foundation for analyzing the behaviour of pension systems under varying conditions.

2.1 The financial market

The price of financial assets plays a critical role in understanding investment risk within CDC schemes. For analytical simplicity, we consider a portfolio comprising two asset types: a risk-free asset offering a constant rate of return and a risky asset. The risky asset is modelled using a lognormal distribution, representing a discrete-time adaptation of Black and Scholes (1973) and Merton's (1973) continuous-time model. This approach facilitates the examination of investment dynamics and risk assessment in such schemes.

We consider a discrete-time economy over the time horizon $\mathcal{T} = \{0, 1, 2, \dots, T\}$. The underlying uncertainty is modelled using a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\mathbb{P}$ denotes the physical measure. This probability space is equipped with a filtration $\mathbb{F} = \{\mathcal{F}_t\}_{t \in \mathcal{T}}$, satisfying the usual conditions.

The risk-free asset price process is denoted by $P = \{P_t\}_{t \in \mathcal{T}}$ and evolves according to the following dynamics:

$$P_t = P_{t-1} \exp(r),$$

where r represents the constant (annualized) risk-free rate of interest. For convenience and without loss of generality, we set the initial price $P_0 = 1$.

The risky asset price process is represented by $S = \{S_t\}_{t \in \mathcal{T}}$. Its dynamics are given by

$$S_t = S_{t-1} \exp \left(\left(\mu - \frac{\sigma^2}{2} \right) + \sigma \epsilon_t \right),$$

where $\epsilon = \{\epsilon_t\}_{t \in \mathcal{T}}$ is a sequence of independent and identically distributed standard normal random variables under $\mathbb{P}$. The parameters μ and σ denote the average annual return and volatility of the risky asset, respectively, such that

$$\mathbb{E}[S_t | \mathcal{F}_{t-1}] = S_{t-1} e^\mu.$$

For convenience and without loss of generality, we set the initial price $S_0 = 1$.

The model assumes an annualized risk-free rate of $r = 2\%$, aligning closely with the 25-year average yield on one-year Canadian Treasury Bills, which stands at 2.0493%. The assumed average annual return for the risky asset is set at $\mu = 7\%$, with a volatility parameter of $\sigma = 15\%$. These values are consistent with the historical price returns and volatility of the S&P/TSX 60 index over the past two decades, where the actual average return and volatility were recorded at 6.73% and 15.31%, respectively.

2.2 The mortality model

A critical risk factor in the design of a CDC pension scheme is the survival probability curve, which represents the probability that an individual currently aged x will survive for the next t years, reaching age $x + t$. This probability is denoted by ${}_t p_x \equiv \mathbb{P}(T_x \geq t)$, where T_x is the remaining lifetime random variable.

To model these survival probabilities, we employ the Gompertz (1825) law of mortality, which posits that the instantaneous hazard rate—the rate at which individuals die or the force of mortality—increases exponentially with age. The hazard rate is given by

$$\lambda_x = \frac{1}{b} \exp \left(\frac{x - m}{b} \right),$$

where λ_x is the instantaneous hazard rate at age x , m is the mode of the remaining lifetime distribution (in years), and b is the dispersion coefficient.

Based on the force of mortality above, the survival probability ${}_t p_x$ can then be expressed as

$${}_t p_x = \mathbb{P}(T_x \geq t) = \exp \left[\exp \left(\frac{x - m}{b} \right) \left(1 - \exp \left(\frac{t}{b} \right) \right) \right].$$

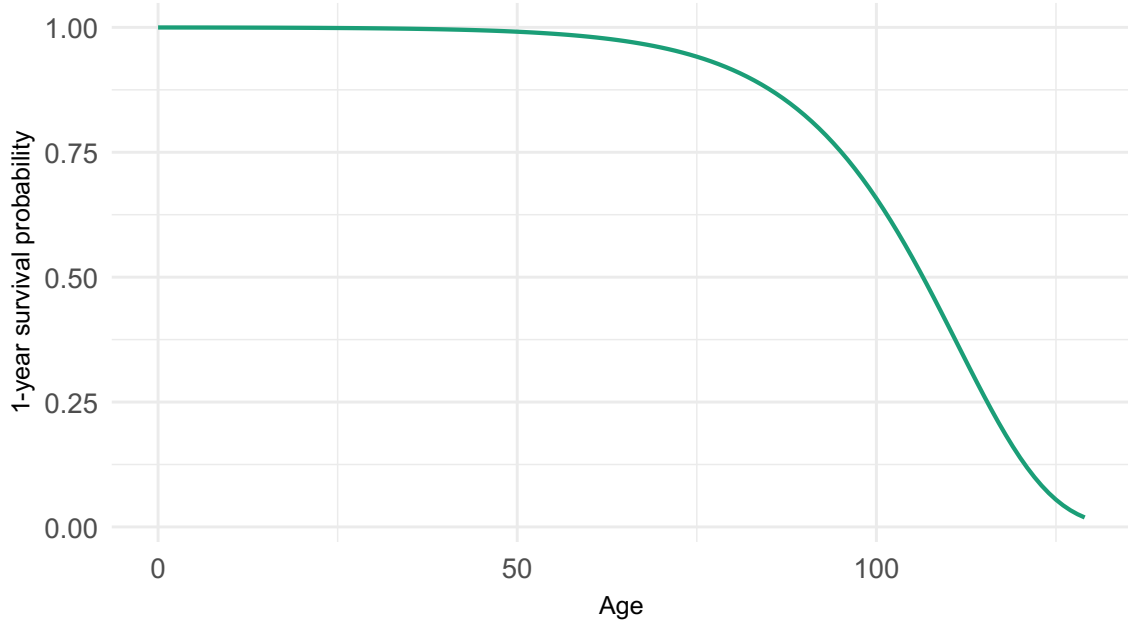


Figure 2.1: One-year survival probabilities under the Gompertz mortality model.

Notes: This figure illustrates the one-year survival probability under the Gompertz mortality model. The figure relies on the following parameters: $m = 78.70589$ and $b = 12.90797$.

We fit the Gompertz mortality model using the R function `nls2`, which employs the Gauss–Newton algorithm to determine non-linear least squares estimates. The model is calibrated using Canadian all-sex mortality data for 2022, for ages 25 and above, sourced from the Human Mortality Database. The resulting parameter estimates are:

$$m = 78.70589 \quad \text{and} \quad b = 12.90797.$$

These estimates are subsequently used to generate mortality rates throughout this report.

Chapter 3

The stylized pension plan design

This chapter outlines the stylized design proposed in this report and examines the adjustments to future benefits. The proposed framework generalizes multiple designs previously explored in the literature, thereby enhancing the potential for benefit smoothing in existing schemes.

We first consider a simplified design comprising only retirees, structured as decumulation-only CDC pension schemes inspired by variable payment life annuities (VPLAs) and group self-annuitization (GSA) schemes. The subsequent section extends this framework to incorporate active members as well.

3.1 Membership profile and asset value dynamics

To introduce our proposed scheme, we first consider a simplified structure for the dynamics of the CDC pension plan. The operational framework shares certain similarities with the newly implemented CDC pension scheme of United Kingdom's Royal Mail Group, among others.

We define $\mathcal{L}_t$ as the set of all surviving members in the plan at time t , such that $k \in \mathcal{L}_t$ if the k^{th} member is alive at time t . To accommodate the inclusion of new members, we introduce the set $\mathcal{L}_t^+$, which represents the scheme's members at time t after accounting for new entrants.

Similarly, we define $\mathcal{A}_t$ as the set of all active members in the plan at time t , and $\mathcal{A}_t^+$ as the set of all active members after incorporating new entrants at time t . We also define $\mathcal{R}_t$ as the set of all retired members in the plan at time t , while $\mathcal{R}_t^+$ represents the set of all retired members, including those who retired at time t .

For convenience, we assume that the stylized scheme disburses benefits at the beginning of each year, while active members contribute to the plan at the same time. Time is measured

in annual increments. Additionally, we incorporate a mechanism for inflation protection by embedding systematic benefit adjustments into the design. Specifically, benefit payments are structured to increase geometrically each year by a factor of e^i , where i denotes the targeted continuously compounded rate of benefit increment. A framework with level benefit payments can be accommodated by setting $i = 0$.

Using standard actuarial notation, let $\ddot{a}_x^h$ represent the actuarial present value of an annuity due with annual payments for a life aged x , defined as

$$\ddot{a}_x^h = \sum_{s=0}^{\infty} {}_s p_x e^{-hs},$$

where h denotes the continuously compounded hurdle rate used to determine the annuity price. The selection of the hurdle rate plays a crucial role in shaping the annual benefits disbursed to retirees—a lower hurdle rate results in reduced initial benefits but enhances the potential for future benefit increases, whereas a higher hurdle rate leads to larger initial benefits at the expense of future adjustments. In this report, we adopt a constant hurdle rate equal to the long-term expected returns of the investment portfolio.

Using the same notation, we define the actuarial value of an annuity due with annual payments that increase geometrically over time as

$$\ddot{a}_x^{h-i} = \sum_{s=0}^{\infty} {}_s p_x e^{-hs} e^{is} = \sum_{s=0}^{\infty} {}_s p_x e^{(i-h)s}.$$

The scheme invests in two asset classes previously introduced: the risky asset S and the risk-free asset P . The allocation between these assets is static over time, with a fraction ω allocated to the risky asset and $1 - \omega$ allocated to the risk-free asset. The continuously compounded rate of return on the asset portfolio at time t is thus given by

$$r_t^{\text{PF}} = \log \left(\omega \frac{S_t}{S_{t-1}} + (1 - \omega) \frac{P_t}{P_{t-1}} \right).$$

Consequently, the hurdle rate associated with this investment portfolio is given by

$$h = \log (\omega e^{\mu} + (1 - \omega) e^r).$$

We define three asset values to distinguish between different stages of the pension plan evolution process.

1. $A(t^-)$ represents the asset value before redistribution to survivors and before new contributions at time t .

2. $A(t)$ denotes the asset value after redistribution to survivors but before new contributions at time t .
3. $A(t^+)$ is the asset value after redistribution to survivors and after new contributions at time t .

Let x_k denote the age of the k^{th} member at the inception of the plan. The minimum age at which individuals are eligible to join the plan—also serving as the entry age for new members from the second year onward—is denoted by ς . Members are assumed to retire at a fixed retirement age R , and the maximum attainable age under the specified mortality model is represented by τ , beyond which survival is assumed to be zero. At inception, the plan is considered to have attained a stationary population distribution, implying the presence of members spanning all ages from ς to τ .

At inception, all members with ages $x_k > \varsigma$ are assumed to enter the plan with an initial asset allocation denoted by $A_k(0)$. The specific value of these initial assets depends on the chosen plan design and will be formally defined in subsequent sections. During their active working years, each member contributes a fixed proportion of their salary to the plan. Salaries are assumed to grow at a constant, continuously compounded rate s . Let $C_k(t)$ denote the contribution made by the k^{th} member at time t . As contributions cease upon retirement, we impose the condition $C_k(t) = 0$ for all $t > R - x_k$, where R denotes the fixed retirement age.

The benefit entitlement of the k^{th} member at time t is denoted by $B_k(t)$. Only retired members receive this benefit payment; for active members, $B_k(t)$ is a notional amount that can be considered earned and funded (although not guaranteed) but not yet payable.

For an individual plan member $k \in \mathcal{L}_t$, the total assets at time t , denoted by $A_k(t)$, evolve according to

$$A_k(t) = (A_k(t-1) + C_k(t-1) - B_k(t-1)) \exp(r_t^{\text{PF}}).$$

Aggregating over all members, the total assets in the plan at time t , before new entrants join, are given by

$$A(t) = \left(A(t-1) + \sum_{j \in \mathcal{L}_{t-1}^+ \setminus \mathcal{L}_{t-1}} A_j(t-1) + \sum_{j \in \mathcal{A}_{t-1}^+} C_j(t-1) - \sum_{j \in \mathcal{R}_{t-1}^+} B_j(t-1) \right) \exp(r_t^{\text{PF}}), \quad (3.1)$$

where $\mathcal{L}_{t-1}^+ \setminus \mathcal{L}_{t-1}$ is the set difference between $\mathcal{L}_{t-1}^+$ and $\mathcal{L}_{t-1}$, which represents new members who joined at time $t-1$. Hence $\sum_{j \in \mathcal{L}_{t-1}^+ \setminus \mathcal{L}_{t-1}} A_j(t-1)$ is the total asset brought in by new entrants at time $t-1$.

Based on the updated asset value $A(t)$ in Equation (3.1), future benefits must be adjusted to reflect changes in the plan’s financial position. In this report, we consider three distinct approaches to implementing these adjustments:

1. Full and immediate recognition of experience in the benefit adjustment at time t .
2. Gradual recognition of experience over the next n years through an indexation mechanism.
3. A hybrid approach that balances immediate and delayed recognition of experience.

3.2 Decumulation-only CDC pension plan design

To incorporate mortality and investment experience adjustments while facilitating intergenerational risk sharing among retirees, we consider a simplified design inspired by VPLAs and GSAs. In this framework, we assume the absence of active members in the plan; that is, $\mathcal{A}_t = \emptyset$ for all t .

In the rest of the section, we apply the approaches introduced in the previous section to the context of a decumulation-only CDC plan. The first approach implements full and immediate recognition of experience in benefit adjustments, updating retiree benefits annually based on the previous year’s investment and mortality experience. While this method ensures rapid alignment between benefits and plan assets, it introduces higher volatility in payments and minimizes intergenerational risk sharing.

The second approach smooths benefit adjustments over a predefined period of n years, by introducing an additional indexation mechanism, reducing short-term volatility at the cost of increased intergenerational risk-sharing and potential inequities across cohorts due to random fluctuations.

The third approach adopts a hybrid mechanism, incorporating a fraction of the experience immediately while deferring the remainder over a fixed number of years. This approach balances the trade-offs between volatility and intergenerational risk sharing, allowing for more controlled benefit adjustments over time.

3.2.1 Initial asset values at plan inception

For convenience, we assume that a member aged R at the plan’s inception contributes an initial endowment of 1,000 to the pool; that is, $A_k(0) = 1,000$ for $x_k = R$. This amount is subsequently converted into an initial retirement benefit using the hurdle rate and the targeted indexation as follows:

$$B_k(0) = \frac{1,000}{\ddot{a}_R^{h-i}} \quad \text{for } x_k = R.$$

For older members at inception (i.e., $x_k > R$), the initial benefit payable $B_k(0)$ is assumed to be the same. The corresponding asset value brought in at inception by members older than age R is the actuarial present value of this benefit, assuming targeted future increases at a continuously compounded rate of i :

$$A_k(0) = B_k(0) \ddot{a}_{x_k}^{h-i} = \frac{1000}{\ddot{a}_R^{h-i}} \ddot{a}_{x_k}^{h-i} \quad \text{for } x_k \geq R.$$

For younger members at inception (i.e., $x_k < R$), the asset value brought into the plan at retirement is assumed to be:

$$A_k(R - x_k) = 1000e^{s(R-x_k)} \quad \text{for } x_k < R.$$

Where s is the annual continuously compounded growth rate for salary. This is consistent with the retirement assets of those age R at inception, assuming identical contribution patterns in real terms over these members' working lives.

3.2.2 Full and immediate recognition of experience

In this approach, we assume that adjustments in benefits at time t uniformly affect all future benefit payments. This approach aligns with existing literature on non-guaranteed pension schemes and exhibits operational similarities to the designs proposed by Piggott et al. (2005) and Qiao and Sherris (2013) in the context of GSA plans, as well as with pooled annuity funds (Donnelly, 2015) and variable annuities (Balter et al., 2020). Under this framework, the benefit of the k^{th} member at time t is determined according to the following rule:

$$B_k(t) = \begin{cases} 0 & \text{if } t < R - x_k \\ \frac{A_k(t)}{\ddot{a}_R^{h-i}} & \text{if } t = R - x_k \\ B_k(t-1) e^i (1 + \alpha(t)) & \text{if } t > R - x_k \end{cases}, \quad (3.2)$$

where $\alpha(t) \in [-1, \infty)$ represents the adjustment factor that accounts for the plan's mortality and investment experience.

Here, the benefit for members younger than the retirement age R is set to zero (since they are not in the plan). Upon reaching retirement at time $t = R - x_k$, the initial benefit amount is determined as the first payment of a geometrically increasing life annuity, priced at the hurdle rate h and growing annually by a factor of e^i . This amount is derived by equating the present value of such an annuity at time t to the k^{th} member's assets at retirement (age R). For retirees aged $R+1$ and older at time t , the benefit level evolves as a function of their prior benefit, increasing geometrically by a factor of e^i each year, subject to the adjustment factor $\alpha(t)$.

Under this benefit structure, the adjustment factor $\alpha(t)$ is determined as follows:

$$A(t) = \sum_{j \in \mathcal{R}_t} B_j(t) \ddot{a}_{x_j+t}^{h-i} = \sum_{j \in \mathcal{R}_t} B_j(t-1) e^i (1 + \alpha(t)) \ddot{a}_{x_j+t}^{h-i},$$

leading to

$$\alpha(t) = \frac{A(t)}{\sum_{j \in \mathcal{R}_t} B_j(t-1) e^i \ddot{a}_{x_j+t}^{h-i}} - 1,$$

where $A(t)$ is the total plan assets at time t obtained from Equation (3.1).

This formulation ensures that all updates are fully incorporated into the current benefit, immediately reflecting each year's investment and mortality experience in the annual benefit adjustments.

Importantly, the benefit update defined in Equation (3.2) applies uniformly to all current members, mitigating excessive transfers between different cohorts. Moreover, as new retirees enter the scheme, their benefits are determined independently of past benefit adjustments, ensuring fairness in intergenerational risk sharing.

3.2.3 Gradual recognition of experience over the next n years

This approach introduces a smoothing mechanism by distributing the impact of experience adjustments over the next n payments through the indexation rate. Instead of immediately reflecting all changes in the benefit amount, we modify the rate of indexation applied at time t and the subsequent $n-1$ years. This gradual adjustment aims to reduce benefit volatility while controlling intergenerational risk sharing.

The core principle of this method is to determine an additional indexation rate i_t , applied consistently for the next n payments, ensuring that the present value of future benefits aligns with the available assets. We label this parameter n as the smoothing window length. To formalize this, we introduce a function f that generates expected future benefit payment cashflows, defined as

$$f(\gamma, \delta) = \sum_{\theta=1}^{\gamma-1} \left[e^{\theta i - (\theta-1)h} {}_{\theta-1}p_{x_k+t} \prod_{\phi=1}^{\delta} e^{\min\{\theta, \phi\} i_{t-n+\phi}} \right] + \left[e^{\gamma i - (\gamma-1)h} {}_{\gamma-1}p_{x_k+t} \prod_{\phi=1}^{\delta} e^{\min\{\gamma, \phi\} i_{t-n+\phi}} \right] \ddot{a}_{x_k+t+n-1}^{h-i},$$

where γ refers to the number of years in which future indexation is to be adjusted and δ is defined as number of years of past indexation still in effect at current time. This function consists of three principal components: a discounting factor $e^{-(\theta-1)h}$, a survival probability term ${}_{\theta-1}p_{x_k+t}$, and a sequence of past indexation adjustments that remain applicable.

Utilizing this function, the updated benefit for the k^{th} member under this approach is defined as

$$B_k(t) = \begin{cases} 0 & \text{if } t < R - x_k \\ \frac{A_k(t)}{f(n, n-1)} & \text{if } t = R - x_k \\ B_k(t-1)e^{i+i_{t-n+1}+i_{t-n+2}+\dots+i_t} & \text{if } t > R - x_k \end{cases}, \quad (3.3)$$

where i_t represents the additional indexation rate determined at time t to ensure that the available assets correspond to the present value of future benefits. This indexation rate i_t is derived by solving the following equation:

$$A(t) = \sum_{j \in \mathcal{R}_t} B_j(t-1) f(n, n-1), \quad (3.4)$$

where $A(t)$ represents the total plan assets at time t as determined by Equation (3.1). Since we can only start announcing new indexation rates from time $t = 1$ onwards, we also impose the condition that $i_t = 0$ for all $t < 1$.

For example, by choosing $n = 3$, Equation (3.3) becomes

$$B_k(t) = \begin{cases} 0 & \text{if } t < R - x_k \\ \frac{A_k(t)}{\left[e^{i+i_{t-2}+i_{t-1}} + e^{-h} {}_1p_{x_k+t} e^{2i+i_{t-2}+2i_{t-1}} \right.} & \text{if } t = R - x_k \\ \left. + e^{-2h} {}_2p_{x_k+t} e^{3i+i_{t-2}+2i_{t-1}} \ddot{a}_{x_k+t+2}^{h-i} \right]} & \\ B_k(t-1)e^{i+i_{t-2}+i_{t-1}+i_t} & \text{if } t > R - x_k \end{cases}.$$

In this example, we can clearly see that new members entering the plan at time $t = R - x_k$ are priced based on all the indexation rates announced up to time $t-1$, but they do not have to participate in any experience sharing from prior years, by taking on excess indexation announced at time t .

The key characteristic of this approach is that it minimizes abrupt changes in current retirees' benefits while ensuring a more gradual adjustment for future beneficiaries. This method smooths out fluctuations in benefits by spreading the effect of investment and mortality experience over a period of n years.

Solving for i_t in Equation (3.4) requires numerical root-finding techniques. In this report, we employ Alefeld et al.'s (1995) ACM Algorithm 748, implemented in the R package **pracma** via the **fzero** function, to determine the appropriate indexation rate i_t .

3.2.4 Hybrid adjustment mechanism

To mitigate the potential intergenerational inequities arising from excessive smoothing in the approach introduced in the previous Section 3.2.3, wherein experience adjustments might be deferred disproportionately into the future, a hybrid mechanism is introduced. This approach combines immediate and deferred gradual recognition of experience by integrating a fraction ξ of the full benefit adjustment $\alpha(t)$ directly into $B_k(t)$ at time t , while the remainder is incorporated into the indexation rate and systematically distributed over n years. We label this parameter ξ as the immediate experience recognition rate. The benefit structure is formally defined as follows:

$$B_k(t) = \begin{cases} 0 & \text{if } t < R - x_k \\ \frac{A_k(t)}{f(n, n-1)} & \text{if } t = R - x_k \\ B_k(t-1)e^{i_t + i_{t-n+1} + i_{t-n+2} + \dots + i_t} (1 + \xi\alpha(t)) & \text{if } t \geq R - x_k \end{cases},$$

where i_t is the indexation rate at time t , in addition to previously established adjustments, ensuring that available assets align with the present value of future benefits. This rate satisfies

$$A(t) = \sum_{j \in \mathcal{R}_t} B_j(t-1) f(n, n-1) (1 + \xi\alpha(t))$$

where $A(t)$ is obtained from Equation (3.1) and $\alpha(t)$ is defined as

$$\alpha(t) = \frac{A(t)}{\sum_{j \in \mathcal{R}_t} B_j(t-1) f(n, n-1)} - 1$$

The parameters ξ and n govern the degree of intergenerational risk sharing. The parameter ξ determines the fraction of experience adjustments absorbed immediately by current retirees, whereas n dictates the temporal spread of remaining adjustments over future years and consequently, future cohorts. Notably, setting $\xi = 1$ recovers the classic GSA design, while setting $\xi = 0$ and $n \rightarrow \infty$ aligns with a design similar to that proposed by the Royal Mail Pension Plan in the UK (barring the indexation floor and lump sum components). Intermediate values of ξ and n yield hybrid solutions balancing these two paradigms.

3.3 CDC pension plan design with active members

To extend the framework introduced in the previous section, which facilitates mortality pooling and intergenerational risk sharing among retirees, we now incorporate active members into the scheme, allowing them to participate in the same risk-sharing mechanism.

Active members contribute a fixed percentage of their salary at the beginning of each year of employment, from age ς until retirement at age $R - 1$. These contributions grow annually at a constant continuously compounded rate s . For all other individuals—retirees and future entrants—contribution levels are set to zero; that is, $C_k(t) = 0$ for all $k \notin \mathcal{A}_t^+$. Successive cohorts are assumed to receive identical real salaries, implying equal contributions in real terms across generations. Let $C_k(t)$ denote the contribution made by member k at time t . For an active member, this evolves as $C_k(t) = C_k(t - 1)e^s$. The total contributions made by member k accumulated with interest and adjusted for survival within the pool over the active employment period from $t = \varsigma - x_k$ to $t = R - x_k$ are given by

$$\sum_{y=0}^{R-\varsigma-1} C_k(\varsigma - x_k) e^{sy} \frac{e^{h((R-\varsigma)-y)}}{(R-\varsigma)-y p_{\varsigma+y}} = C_k(\varsigma - x_k) \ddot{a}_{\varsigma:(R-\varsigma)}^{h-s} \frac{e^{(R-\varsigma)h}}{(R-\varsigma)p_{\varsigma}}. \quad (3.5)$$

3.3.1 Initial asset values at plan inception

As before, we assume that a member aged R at the plan's inception contributes an initial endowment of 1,000 to the pool; that is, $A_k(0) = 1,000$ for $x_k = R$. This amount is converted into an initial retirement benefit using the hurdle rate and the targeted indexation:

$$B_k(0) = \frac{1,000}{\ddot{a}_R^{h-i}} \quad \text{for } x_k = R.$$

For older members at inception (i.e., $x_k > R$), the initial benefit payable $B_k(0)$ reflects their lower nominal contributions during their working life (decreasing the benefit by a factor of e^s per year) and the indexation applied since reaching retirement age (increasing the benefit by a factor of e^i per year). If $s = i$, these effects offset each other, resulting in uniform benefits for all retirees at inception. If $s \neq i$, the initial benefit follows the recursion

$$B_{k+1}(0) = B_k(0) e^{(i-s)(x_{k+1}-x_k)} \quad \text{for } x_k > R.$$

The corresponding asset value brought in at inception by members older than age R is the actuarial present value of this benefit, assuming targeted future increases at a continuously compounded rate of i :

$$A_k(0) = B_k(0) \ddot{a}_{x_k}^{h-i} = \frac{1000}{\ddot{a}_R^{h-i}} e^{(i-s)(x_k-R)} \ddot{a}_{x_k}^{h-i} \quad \text{for } x_k \geq R.$$

For younger members at inception (i.e., $x_k < R$), the initial asset value is determined as the accumulated value of their contributions. To compute this, we first establish the contribution level. Since a member aged R at inception brings 1,000 to the pool, a younger member's projected asset value at retirement, assuming identical real contribution patterns,

is adjusted at rate s :

$$A_k(R - x_k) = 1000e^{s(R-x_k)} \quad \text{for } x_k < R. \quad (3.6)$$

By equating Equations (3.5) and (3.6), we obtain

$$\begin{aligned} C_k(\varsigma - x_k) \ddot{a}_{\varsigma:(R-\varsigma)}^{h-s} \frac{e^{(R-\varsigma)h}}{(R-\varsigma)p_\varsigma} &= 1000e^{s(R-x_k)} \\ \Rightarrow C_k(\varsigma - x_k) &= \frac{1000e^{s(R-x_k)} (R-\varsigma)p_\varsigma}{\ddot{a}_{\varsigma:(R-\varsigma)}^{h-s} e^{(R-\varsigma)h}}. \end{aligned}$$

The amount brought in by active members at inception (i.e., $\varsigma < x_k < R$) is then the accumulated value of this contribution. At time $t = 0$, before the new contributions are made, accounting for contributions increasing at rate s , accumulating with interest at rate h , and adjusting for survivorship, it is given by

$$A_k(0) = C_k(\varsigma - x_k) \ddot{a}_{\varsigma:x_k-\varsigma}^{h-s} \frac{e^{h(x_k-\varsigma)}}{x_k-\varsigma p_\varsigma}.$$

This account value is converted into an initial notional benefit level, with payments assumed to commence at age R , with targeted increases annually at a continuously compounded rate i , both during the active service period and after retirement, such that

$$B_k(0) = \frac{A_k(0)}{\ddot{a}_R^{h-i} e^{-(h-i)(R-x_k)} R-x_k p_{x_k}}.$$

Finally, after new contributions are made at time $t = 0$, the benefit for active members is updated as follows:

$$B_k(0^+) = B_k(0) + \frac{C_k(0)}{\ddot{a}_R^{h-i} e^{-(h-i)(R-x_k)} R-x_k p_{x_k}} \quad \text{for } \varsigma \leq x_k < R.$$

In this section, we extend the three approaches previously introduced to incorporate active members. Given that the underlying objectives regarding benefit disbursements and contribution structures remain consistent across all approaches, the initial benefit levels and asset valuations at inception are identical.

3.3.2 Full and immediate recognition of experience

Consistent with the framework outlined in Section 3.2.2, this approach assumes that benefit adjustments at time t apply uniformly to all future benefit payments. Under this paradigm, the benefit entitlement of the k^{th} member at time t who participated in the plan at time

$t - 1$ evolves according to the following update mechanism:

$$B_k(t) = B_k(t - 1) e^i (1 + \alpha(t)), \quad (3.7)$$

where i is the targeted indexation rate and $\alpha(t)$ represents the adjustment factor that accounts for deviations in mortality and investment experience.

The adjustment factor is determined by solving

$$A(t) = (1 + \alpha(t)) \times \left[\sum_{j \in \mathcal{A}_t} B_j(t - 1^+) e^i \ddot{a}_R^{h-i} e^{-(h-i)(R-x_j-t)} {}_{R-x_j-t}p_{x_j+t} + \sum_{j \in \mathcal{R}_t^+} B_j(t - 1) e^i \ddot{a}_{x_j+t}^{h-i} \right],$$

which yields

$$\alpha(t) = \frac{A(t)}{e^i \left[\sum_{j \in \mathcal{A}_t} B_j(t - 1^+) \ddot{a}_R^{h-i} e^{-(h-i)(R-x_j-t)} {}_{R-x_j-t}p_{x_j+t} + \sum_{j \in \mathcal{R}_t^+} B_j(t - 1) \ddot{a}_{x_j+t}^{h-i} \right]} - 1.$$

Here, $A(t)$ denotes the total plan assets at time t as defined in Equation (3.1). This formulation ensures that all realized experience is immediately incorporated into the benefits, thereby fully reflecting investment and mortality experience in annual adjustments.

For active members, benefit entitlements are further adjusted to account for new contributions made at time t :

$$B_k(t^+) = B_k(t) + \frac{C_k(t)}{\ddot{a}_R^{h-i} e^{-(h-i)(R-x_k-t)} {}_{R-x_k-t}p_{x_k+t}} \quad \text{for } \varsigma \leq x_k + t < R.$$

A key feature of this approach is the uniform application of the benefit adjustments across all current members. Furthermore, as new members enter the plan, their benefits are determined independently of prior benefit adjustments, ensuring equitable risk sharing across cohorts.

3.3.3 Gradual recognition of experience over the next n years

In line with the methodology employed in the decumulation-only case, this approach incorporates a smoothing mechanism by distributing the impact of experience adjustments over a predetermined period of n years through modifications to the indexation rate. Rather than immediately reflecting all changes in the benefit amount, this method systematically adjusts the rate of indexation applied at time t and for the subsequent $n - 1$ years. The primary objective is to reduce benefit volatility while maintaining equitable intergenerational risk sharing.

The core principle underlying this approach is the determination of an additional indexation rate, i_t , which is applied consistently over the next n years to ensure that the present value of future benefits remains aligned with the plan's available assets. To formalize this, we define a function g that generates expected future benefit payment cashflows, for active members, similar to $f(\gamma, \delta)$ in the decumulation-only case:

$$g(\gamma, \delta) = \sum_{\theta=1}^{\gamma-1} \left[e^{-(R+\theta-1-x_k-t)(h-i)} {}_{R+\theta-1-x_k-t}P_{x_k+t} \prod_{\phi=1}^{\delta} e^{\min\{\theta, \phi\}i_{t-n+\phi}} \right] + \left[e^{-(R+\gamma-1-x_k-t)(h-i)} {}_{R+\gamma-1-x_k-t}P_{x_k+t} \prod_{\phi=1}^{\delta} e^{\min\{\gamma, \phi\}i_{t-n+\phi}} \right] \ddot{a}_{R+\gamma-1-x_k-t}^{h-i}.$$

This function comprises three principal components: a discounting factor $e^{-(R+\theta-1-x_k-t)(h-i)}$, a survival probability term ${}_{R+\theta-1-x_k-t}P_{x_k+t}$, and a sequence of past indexation adjustments that remain applicable. Notably, compared to the decumulation-only case, these benefit payments are deferred by an additional $R - x_k - t$ years, as active members only commence receiving benefits upon reaching retirement age R .

Utilizing this function, the benefit amount for the k^{th} member evolves under this approach as follows:

$$B_k(t) = B_k(t-1^+)e^{i^+i_{t-n+1}+i_{t-n+2}+\dots+i_t},$$

with a further adjustment for active members to account for new contributions:

$$B_k(t^+) = \begin{cases} B_k(t) + \frac{C_k(t)}{g(n, n-1)} & \text{if } t < R - x_k \\ B_k(t) & \text{if } t \geq R - x_k \end{cases}, \quad (3.8)$$

where i_t represents the additional indexation rate determined at time t to ensure that the present value of future benefits aligns with available assets. The indexation rate i_t is obtained by solving the following equilibrium equation:

$$A(t) = \sum_{j \in \mathcal{A}_t} B_j(t-1^+)g(n, n-1) + \sum_{j \in \mathcal{R}_t^+} B_j(t-1)f(n, n-1), \quad (3.9)$$

where $A(t)$ denotes the total plan assets at time t , as specified in Equation (3.1).

A distinguishing feature of this approach is its ability to dampen abrupt fluctuations in retirees' benefits while facilitating a more measured adjustment process for all members in plan. By distributing the effects of investment and mortality experience over multiple years, this methodology is intended to enhance stability in benefit payments.

As before, solving for i_t in Equation (3.9) necessitates numerical root-finding techniques. Once again, we employ Alefeld et al.'s (1995) ACM Algorithm 748, implemented in the R package `pracma` via the `fzero` function, to determine the optimal indexation rate i_t .

3.3.4 Hybrid adjustment mechanism

Under this most generalized approach, we extend the framework previously introduced for a decumulation-only plan consisting solely of retirees. To mitigate the risk of excessive smoothing, which could disproportionately transfer the impact of negative experience to future generations, we integrate elements of both approaches above. Specifically, a fraction ξ of the experience adjustment is recognized immediately, while the remaining $1 - \xi$ portion is gradually incorporated over the next n years through an additional indexation rate.

The evolution of benefits under this approach follows

$$B_k(t) = (1 + \xi\alpha(t)) B_k(t - 1^+) e^{i + i_{t-n+1} + i_{t-n+2} + \dots + i_t},$$

where $B_k(t - 1^+)$ is obtained from Equation (3.8), and i_t represents the additional indexation rate determined at time t to ensure that the present value of future benefits remains consistent with available assets. The indexation rate i_t is derived by solving the following equation:

$$A(t) = (1 + \xi\alpha(t)) \left[\sum_{j \in \mathcal{A}_t} B_j(t - 1^+) g(n, n - 1) + \sum_{j \in \mathcal{R}_t^+} B_j(t - 1) f(n, n - 1) \right], \quad (3.10)$$

where $A(t)$ represents the total plan assets at time t , as specified in Equation (3.1). The adjustment factor $\alpha(t)$ is defined as

$$\alpha(t) = \frac{A(t)}{\sum_{j \in \mathcal{A}_t} B_j(t - 1^+) g(n, n - 1) + \sum_{j \in \mathcal{R}_t^+} B_j(t - 1) f(n, n - 1)} - 1.$$

The numerical solution for i_t follows the same procedure outlined in previous sections, employing Alefeld et al.'s (1995) ACM Algorithm 748, implemented via the `fzero` function in the R package `pracma`.

This hybrid approach ensures a balanced trade-off between immediate recognition of plan experience and a controlled smoothing process, thereby promoting stability while maintaining intergenerational fairness.

This hybrid framework serves as a comprehensive generalization of all the approaches discussed thus far. By appropriately selecting the parameters, immediate experience recogni-

tion rate ξ and smoothing window length n , we can recover each of the previously introduced methods as well as certain well-known CDC pension schemes documented in the literature.

For instance, setting $\xi = 1$ fully recovers the first approach, which extends the traditional GSA plan framework to include active members. Alternatively, choosing $\xi = 0$ and $n = 1$ yields results nearly identical to the first approach with immediate experience recognition and minimal intergenerational risk-sharing, except that experience adjustments are deferred by one year. Setting $\xi = 0$ recovers the second approach, which, when further extended to $n \rightarrow \infty$, closely resembles the CDC scheme adopted by UK's Royal Mail Group. Other intermediate choices of ξ and n allow for flexible hybrid solutions that strike a balance between immediate and gradual experience recognition, accommodating a range of intergenerational risk-sharing preferences.

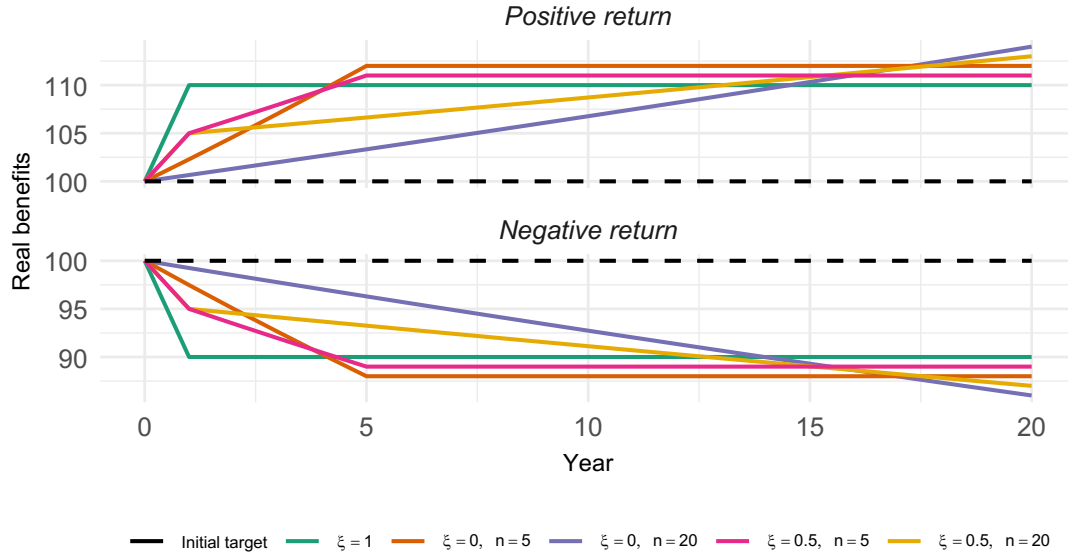


Figure 3.1: Impact of positive and negative returns on future benefit expectations in real terms.

Notes: This figure shows the expected benefit streams in real terms after a large positive return of $h + 10\%$ (top panel) and a large negative return of $h - 10\%$ (bottom panel) during the first year. The black dashed line corresponds to the benefit expectation in real terms at the plan's inception. The coloured lines are the expected benefit streams in real terms arising from the first year's experience.

Figure 3.1 illustrates the effect of an unexpected return shock in year 1 on the evolution of expected real benefit payments over the first 20 years, under various combinations of the design parameters ξ and n . The top panel displays benefit trajectories following a one-time positive shock—a return 10% above the expected rate h —while the bottom panel shows the impact of a 10% shortfall relative to h . In both panels, the black dashed line represents the initial benefit expectation at plan inception, serving as a benchmark. The coloured

lines show the updated expected benefit trajectories after incorporating the year 1 return experience.

Under designs with full immediate recognition of experience ($\xi = 1$, GSA-type), the entire shock is absorbed in year 1, leading to a permanent upward (top panel) or downward (bottom panel) shift in benefit levels, which then remains constant over time. In contrast, designs with full smoothing ($\xi = 0$) distribute the impact gradually over n years. For instance, when $n = 20$, the benefit stream adjusts incrementally, with the change compounding over time. Intermediate designs (e.g., $\xi = 0.5$) blend these behaviours: increasing ξ results in a larger immediate adjustment but a slower subsequent drift, while reducing n accelerates the reversion to a flat trajectory by concentrating the smoothing period into fewer years (e.g., $n = 5$). The bottom panel mirrors these patterns in the opposite direction, reflecting how the design responds to adverse shocks.

Chapter 4

Evolution of assets, adjustments, and benefits

This chapter presents the simulation study undertaken to evaluate the various approaches for implementing benefit adjustments as outlined in Chapter 3. We examine the distribution of total plan assets $A(t)$, the distributions of the adjustment mechanisms $\alpha(t)$ and i_t , and, in turn, the resulting distribution of benefit payments $B(t)$. Particular attention is given to the influence of the plan design parameters, smoothing window length n and the immediate experience recognition rate ξ , in order to illustrate how these parameters jointly govern the dynamic behaviour of the adjustment mechanisms and the evolution of benefits over time.

4.1 Simulation setting

To evaluate the outcomes under the proposed CDC plan design, we establish a set of fixed design parameters, asset return assumptions, and membership pool characteristics. We then conduct 25,000 simulations over a 100-year horizon to capture a broad range of potential outcomes.

For the design parameters n and ξ , which are relevant for the gradual recognition of experience approach of Section 3.2.3 and 3.3.3 as well as the hybrid adjustment mechanism approach of Section 3.2.4 and 3.3.4, we consider a comprehensive grid. The smoothing window length parameter n is varied across the values 2, 3, 4, 5, 10, 15, 20, 25, 50, and 100 years. For each value of n , results are compared under three different levels of the immediate experience recognition rate parameter: $\xi = 0.25$, $\xi = 0.50$, and $\xi = 0.75$. It is important to recall that setting $\xi = 0$ is equivalent to the full and immediate recognition of experience approach, whereas $\xi = 1$ corresponds to the gradual recognition of experience approach. Throughout all simulations, we maintain a constant target rate of benefit increase of $i = 0.02$.

The asset model assumes a static portfolio allocation with $\omega = 0.50$, meaning that 50% of the plan's portfolio is allocated to a safe, bond-like asset P , and the remaining 50% to a risky, equity-like asset S . The continuously compounded risk-free rate is fixed at $r = 0.02$, which also serves as the expected return on the safe asset P . The risky asset S is assumed to yield an expected return of $\mu = 0.07$ with a volatility of $\sigma = 0.15$, resulting in an implied hurdle rate of about $h = 0.0453$.

For the membership pool, we assume that at plan inception, a member aged $R = 65$ brings precisely \$1000 into the collective plan assets. Members are assumed to experience a fixed annual salary growth rate of $s = 0.02$, aligned with the target rate of benefit increase. The plan is assumed to start in a stationary state, with membership ages ranging from the minimum entry age $\varsigma = 30$ to the terminal age $\tau = 130$. To maintain a stable pool size of approximately 5,000 individuals (4,990 at inception), we assume the annual entry of 116 new members aged 30. In the case of a decumulation-only CDC plan, the inflow is adjusted to 84 new members aged 65 annually, matching the expected number of survivors to age 65 from an initial cohort of 116 members aged 30. This configuration results in a reduced but still sufficiently large pool of approximately 1,250 members (1,272 at inception). The Gompertz mortality model parameters are set to $m = 78.70589$ and $b = 12.90797$.

This membership structure leads to a total of 100 cohorts (or 65 in the decumulation-only scenario) being present in the plan at any given time, though some may have no surviving members. For the sake of convenience, we focus on a representative sample of 35 cohorts. Specifically, we designate the cohort aged $x_k = R = 65$ at inception as Cohort 1, with subsequent cohorts indexed by decreasing age such that Cohort 2 corresponds to those aged 64 at inception, and so on, until Cohort 35 corresponds to those aged 31 at inception. These cohorts are selected because they remain in the plan for their full lifetimes within the 100-year simulation horizon, allowing a thorough investigation of intergenerational dynamics and potential implicit wealth transfers between cohorts.

4.2 Results for the full and immediate recognition of experience approach

We begin by examining results under our first proposed approach, which corresponds to the immediate recognition of mortality and investment experience. In this section, we present three key elements: the evolution of total plan assets, the distribution of the adjustment factor $\alpha(t)$, and the distribution of benefits in real terms received by several cohorts.

Given our modelling assumptions, we expect total plan assets to grow geometrically at a rate of $i = 0.02$, which aligns with both the assumed rate of salary growth and the target rate of benefit increases. This theoretical expectation is reflected in Figure 4.1, which shows the funnels of doubt for the asset trajectories under both the full-plan case with active

members (right panel) and the decumulation-only CDC plan design (left panel). Since the pool size is much smaller in the case of the decumulation-only CDC design, we start with lower total assets, and hence the smaller absolute dispersion is seen in total plan assets, although the growth rate in both cases remains the same.

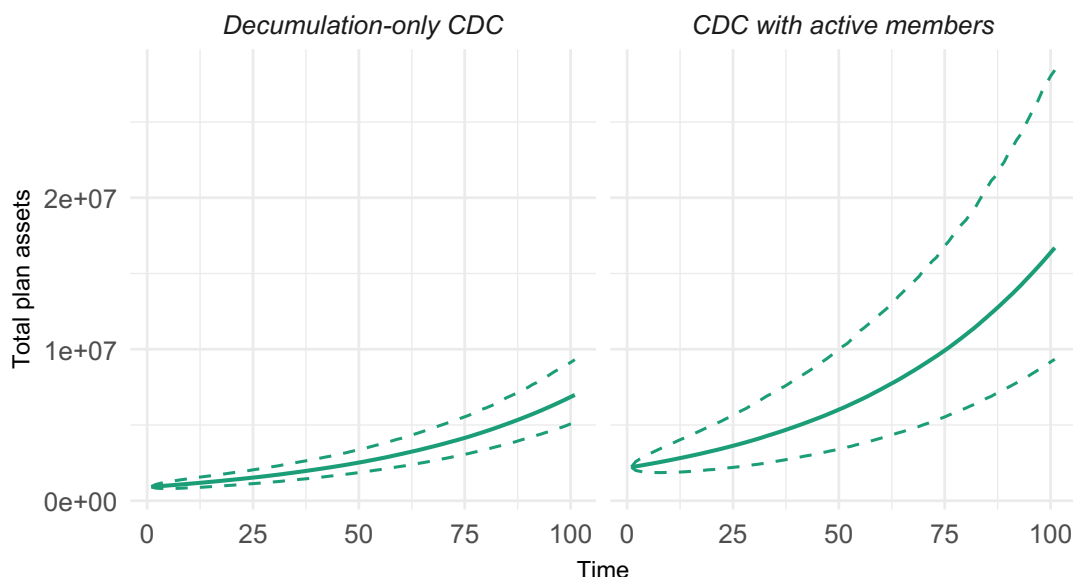


Figure 4.1: Total plan assets for the full and immediate recognition of experience approach.

Notes: This figure depicts the evolution of total plan assets for the full and immediate recognition of experience approach, with the left panel corresponding to the decumulation-only plan and the right panel to the CDC plan with active members. The solid line represents the average, and the dashed lines are the 5th and 95th percentiles of the distribution. We expect them to grow exponentially at a rate of 0.02, which is the assumed salary growth rate and target rate of benefit increase.

We also anticipate that the adjustment factor $\alpha(t)$ will be zero on average each year, as the asset and liability paths are expected to remain aligned on average. Since the full and immediate recognition approach recognizes all mortality and investment experience immediately, and there is no smoothing involved, the presence of active members does not make a difference in the average or the dispersion of $\alpha(t)$. Although one would assume that a larger pool size in the design with active members should lead to less dispersion in values of $\alpha(t)$, Figure 4.2 shows that it remains the same in both cases, albeit the dispersion is marginally higher for the decumulation only case. This is due to the fact that having more than 1,000 members in the pool achieve maximum mortality pooling possible in this type of designs, and the dispersion in $\alpha(t)$ is primary driven by investment experience, hence we do not see much impact from different pool sizes.

Finally, Figure 4.3 presents the funnel of doubt for benefits in real terms (benefit amounts discounted to inception using the target benefit growth rate $i = 0.02$) to selected cohorts under the current approach. This visualization highlights the intercohort variability and the

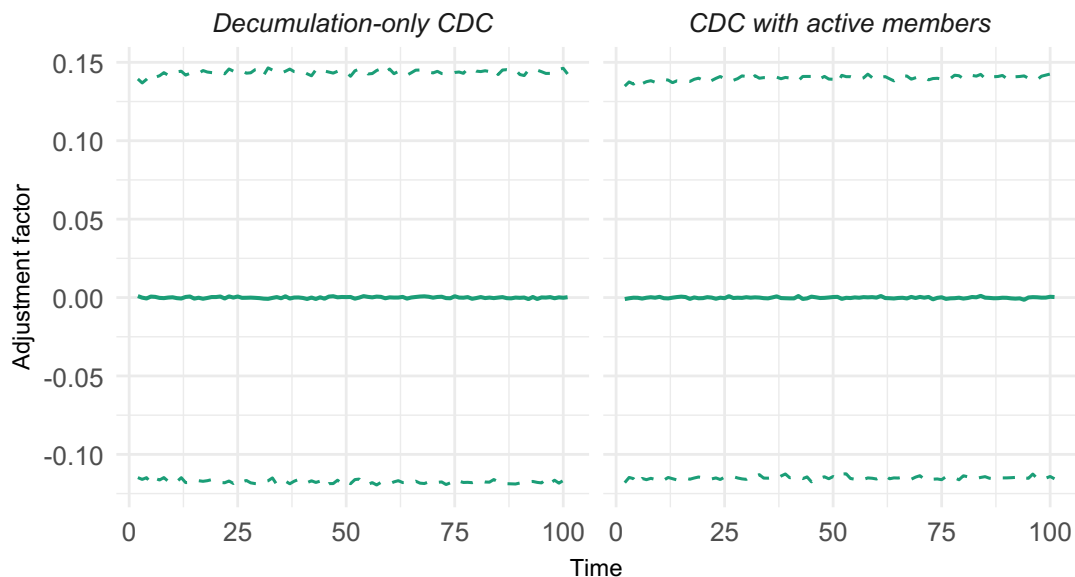


Figure 4.2: **Distribution of adjustment factor $\alpha(t)$ for the full and immediate recognition of experience approach.**

Notes: This figure illustrates the distribution of the adjustment factor $\alpha(t)$ for the full and immediate recognition of experience approach, with the left panel corresponding to the decumulation-only plan, and the right panel to the plan including active members. The solid line represents the average, and the dashed lines are the 5th and 95th percentiles of the distribution.

influence of plan design—namely, whether active members are included. Specifically, the variability in benefit amounts between cohorts is considerably greater when active members are present, as opposed to the decumulation-only setting, where variation between cohorts is minimal.

This discrepancy arises because, in the CDC plan with active members, the presence of contributing members who bear collective risks despite not receiving immediate benefit payments, provides a buffer that stabilizes the plan in the short term. This leads to lower dispersion in outcomes for earlier cohorts, such as Cohort 1, compared to the decumulation-only plan. However, each successive cohort in the plan with active members remains in the plan for a longer duration, allowing benefit adjustments—although centred around zero on average—to accumulate over time. Consequently, cohorts exposed to the adjustment mechanism for 100 years experience greater variability in outcomes than those only exposed for 30 years, thereby contributing to the broader long-term dispersion observed under the CDC with active members. This effect, arising from the difference in participation time between cohorts, is absent in the decumulation-only CDC plan, hence there is no inter-cohort variability. Nevertheless, across both plan types, the average benefit trajectory remains similar, in line with theoretical expectations.

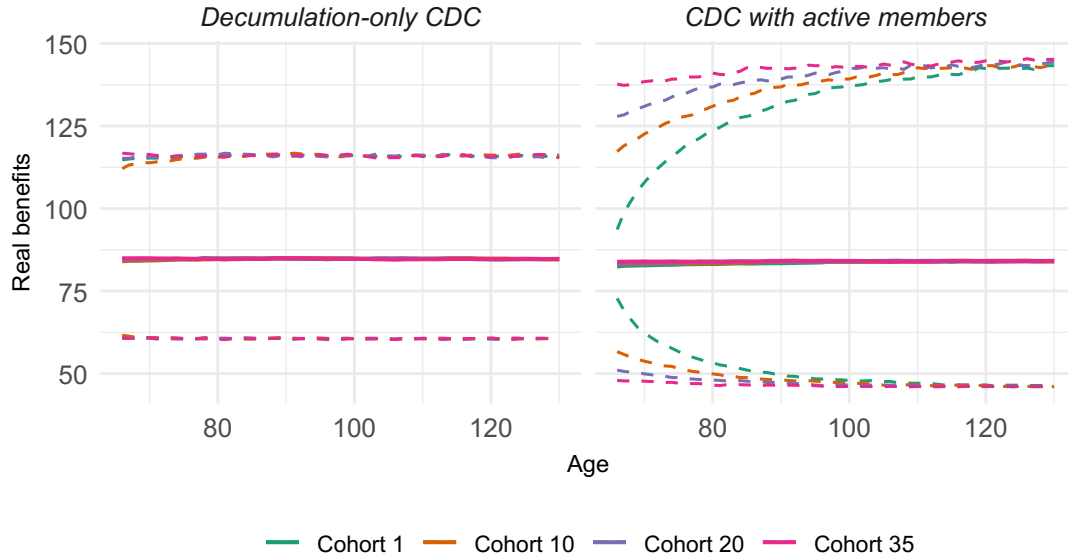


Figure 4.3: Distribution of benefits in real terms for the full and immediate recognition of experience approach.

Notes: This figure shows the distribution of benefits in real terms for the full and immediate recognition of experience approach, with the left panel corresponding to the decumulation-only plan, and the right panel to the plan including active members. The solid line represents the average, and the dashed lines are the 5th and 95th percentiles of the distribution.

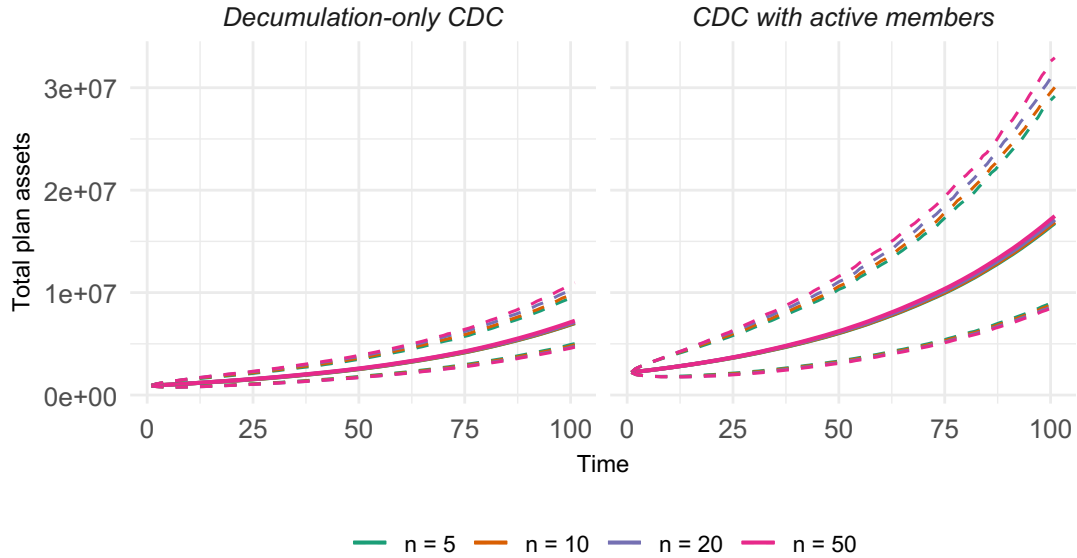


Figure 4.4: Total plan assets for the gradual recognition of experience approach.

Notes: This figure depicts the evolution of total plan assets for the gradual recognition of experience approach for various choices of the design parameter n , with the left panel corresponding to the decumulation-only plan, and the right panel to the plan including active members. The solid lines represent the average, and the dashed lines are the 5th and 95th percentiles of the distribution.

4.3 Results for the gradual recognition of experience approach

We now proceed in a similar fashion to the previous section and examine the outcomes under our second design, which entails the gradual recognition of investment and mortality experience over a fixed number of years n . This mechanism serves to smooth benefit payments during retirement and thereby reduce the volatility of benefits. We present three components: the evolution of plan assets, the distribution of the excess indexation rate i_t for various choices of the smoothing parameter n , and the impact of both n and the presence of active members on real benefit outcomes.

Similar to the first approach, the total plan assets exhibit broadly similar distributional characteristics. Again, we observe a wider spread in the CDC design with active members than in the decumulation-only CDC scheme. This difference arises primarily due to the disparity in initial plan assets and differences in membership pool sizes. Notably, as the smoothing window parameter n increases—meaning that experience is recognized over a longer horizon—the dispersion of total assets in the plan also increases. This occurs because the system may either build up a notional “surplus,” or delay the recognition of a notional “deficit.” These deferred adjustments give rise to a broader range of possible outcomes for total plan assets, as seen in Figure 4.4.

Just as the adjustment factor $\alpha(t)$ in the first approach had an expected long-run average of zero, we anticipate that the excess indexation i_t —applied on top of all previously granted indexation—should also average zero in the long run. Moreover, we expect the variance of i_t to decline as the smoothing parameter n increases. This trend is evident in Figure 4.5. When n is small, experience must be recognized over a shorter time frame, making i_t more susceptible to short-term shocks in investment returns and mortality. In contrast, a longer smoothing window allows for more gradual adjustments and hence a more stable indexation process. The marginal benefit of increasing n diminishes beyond a certain point; for instance, the reduction in volatility when moving from $n = 5$ to $n = 10$ is substantial, while changes from $n = 20$ to $n = 50$ are relatively minor.

Next, we examine the distribution of benefits in real terms received by retirees under the second approach. Figure 4.6 shows the funnel of doubt for benefit trajectories across multiple cohorts, assuming a fixed smoothing window length parameter of $n = 25$. We observe similar patterns to those noted under the full and immediate recognition of experience approach in Figure 4.3, owing to the same underlying mechanisms discussed previously for the plan with active members. However, under the decumulation-only CDC design, we now observe cohort-specific differences in dispersion—unlike the uniform pattern in Figure 4.3. This variation arises from the smoothing process itself: earlier cohorts, particularly those entering during the first $n - 1$ years, benefit from greater short-term stability as the plan

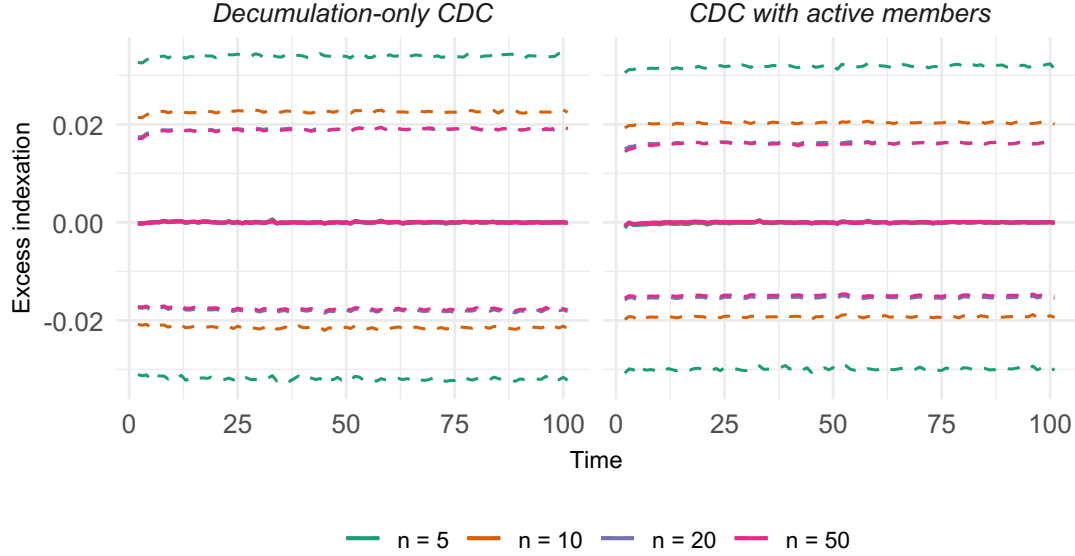


Figure 4.5: **Distribution of excess indexation i_t for the gradual recognition of experience approach.**

Notes: This figure illustrates the distribution of excess indexation i_t for the gradual recognition of experience approach for various choices of the smoothing parameter n , with the left panel corresponding to the decumulation-only plan, and the right panel to the plan including active members. The solid lines represent the average, and the dashed lines are the 5th and 95th percentiles of the distribution.

gradually incorporates the full experience from Year 1. It takes n years to fully recognize these cumulative effects, during which time the plan adjusts to built-up notional surpluses or deficits, eventually reaching a steady-state level of variability in the benefit amounts in real terms.

In Figure 4.7, we fix the cohort to the first one and vary n to assess how different smoothing horizons influence benefit outcomes. As expected, larger values of n result in more gradual benefit changes over time. While this facilitates more stable benefit payments in the short term by reducing the volatility for earlier cohorts, it increases the degree of intergenerational risk sharing, thereby leading to a wider range of eventual outcomes. Conversely, smaller values of n allow for quicker adjustment to realize experience, reducing long-term dispersion. The contrast between the CDC plan with active members and the decumulation-only design again stems from the same reason as explained earlier for the first approach.

4.4 Results for the hybrid adjustment mechanism approach

As in the preceding section, we analyze three key metrics: the trajectory of total plan assets, the distribution of excess indexation i_t , and the benefits in real terms made to retirees. However, unlike in the second approach, this analysis involves two key parameters: the smoothing window length n and the immediate experience recognition rate ξ . Consequently,

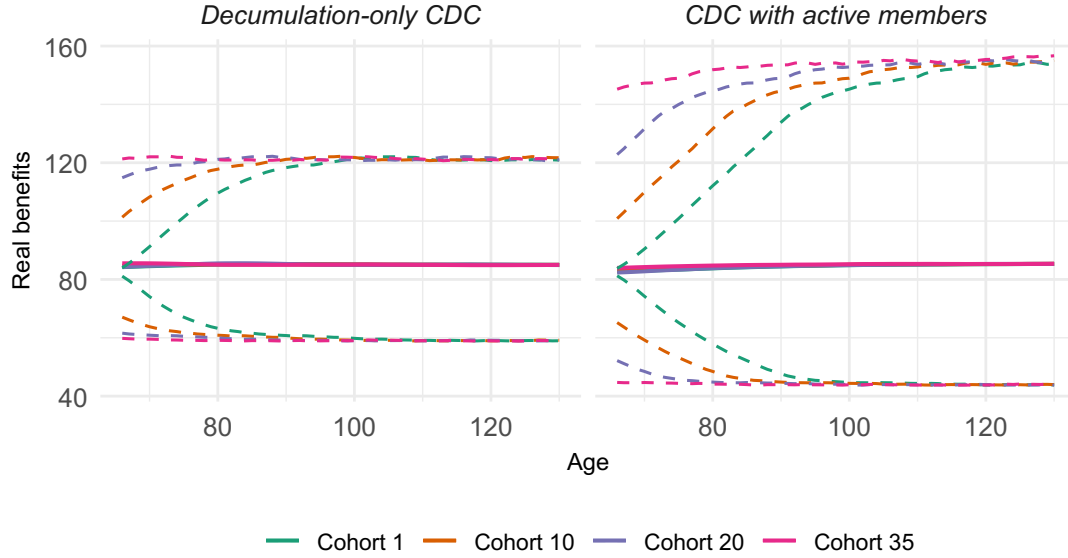


Figure 4.6: **Distribution of benefits in real terms for the gradual recognition of experience approach for $n = 25$.**

Notes: This figure shows the distribution of benefits in real terms for the gradual recognition of experience approach for $n = 25$, with the left panel corresponding to the decumulation-only plan, and the right panel to the plan including active members. The solid lines represent the average, and the dashed lines are the 5th and 95th percentiles of the distribution.

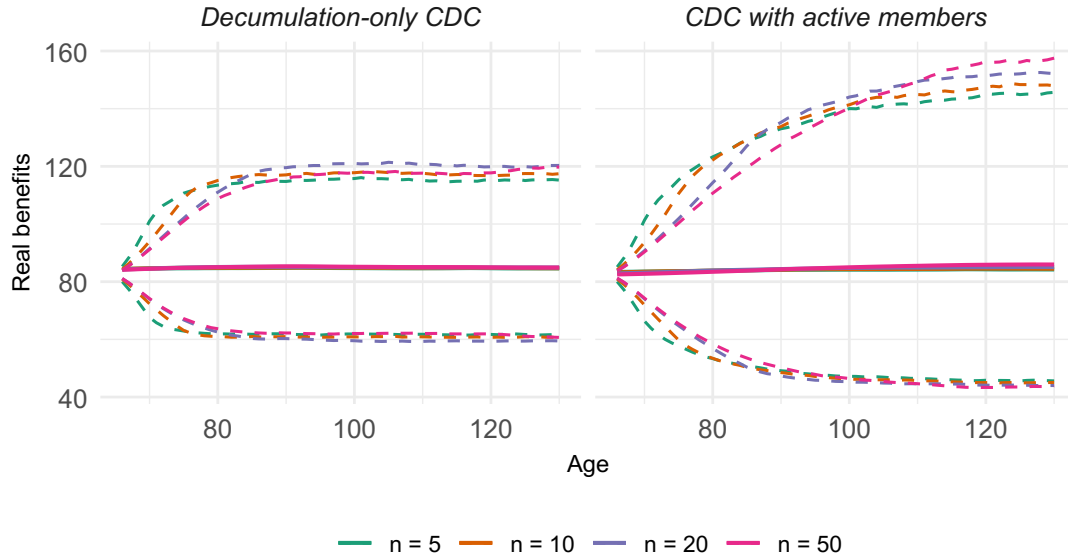


Figure 4.7: **Distribution of benefits in real terms for the gradual recognition of experience approach for varying n .**

Notes: This figure shows the distribution of real benefit amounts under for the gradual recognition of experience approach for Cohort 1, across different values of n , with the left panel corresponding to the decumulation-only plan, and the right panel to the plan including active members. The solid lines represent the average, and the dashed lines are the 5th and 95th percentiles of the distribution.

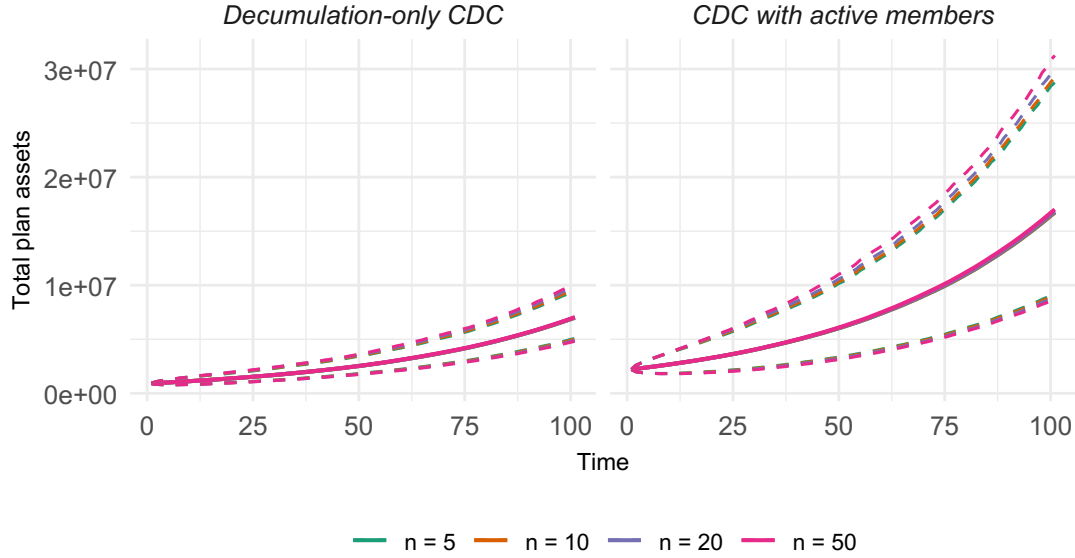


Figure 4.8: **Total plan assets for the hybrid adjustment mechanism approach for varying n and a constant $\xi = 0.50$.**

Notes: This figure shows the evolution of total plan assets for the hybrid adjustment mechanism approach for a range of n values, keeping $\xi = 0.50$ fixed, with the left panel corresponding to the decumulation-only plan, and the right panel to the plan including active members. The solid lines represent the average, and the dashed lines are the 5th and 95th percentiles of the distribution.

we present two complementary views when evaluating the distribution of excess indexation i_t : one where n is held constant and ξ varies, and another where we vary n while holding ξ fixed. We do not include results for $\alpha(t)$ here, as its definition and behaviour mirror that in the first approach. Likewise, we omit plots showing the influence of ξ on total plan assets and benefits in real terms for a fixed cohort, as these remain consistent with the dynamics observed under the second approach.

As with the gradual recognition of experience approach, increasing the smoothing parameter n results in greater dispersion of total plan assets. Figure 4.8 confirms this, though the magnitude of dispersion is more subdued than in the second approach. This moderation arises from setting ξ to 0.50 in the hybrid approach—only half of the investment and mortality experience is deferred, with the remainder being recognized immediately. This limits the range of possible outcomes, thereby reducing the volatility in possible plan asset trajectories. The difference between the CDC plan with actives and the decumulation-only version persists for the same reasons discussed earlier.

Consistent with the gradual recognition of experience approach, the excess indexation i_t is assumed to have an average of zero across all simulations in each year, and its variability increases with the length of the smoothing window length n . This is evident in Figure 4.9, which reports the distribution of i_t across different n values, holding ξ at 0.50.

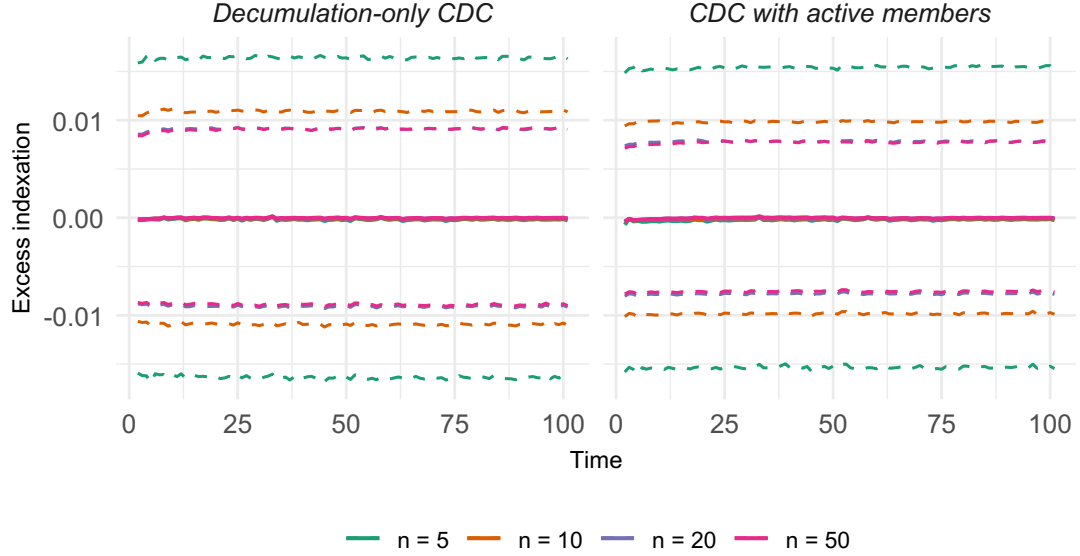


Figure 4.9: **Distribution of excess indexation i_t for the hybrid adjustment mechanism approach for varying n and a constant $\xi = 0.50$.**

Notes: This figure presents the distribution of excess indexation i_t for the hybrid adjustment mechanism approach for different values of the smoothing window n while keeping $\xi = 0.50$ fixed, with the left panel corresponding to the decumulation-only plan, and the right panel to the plan including active members. The solid lines represent the average, and the dashed lines are the 5th and 95th percentiles of the distribution.

Compared to the second approach where $\xi = 0$, the dispersion of i_t here is noticeably lower—approximately half—since only a portion of the experience is deferred and the rest is incorporated immediately. This mixed recognition approach dampens the extremities of experience shock propagation.

Figure 4.10 illustrates the sensitivity of i_t to different levels of ξ , with the smoothing parameter fixed at $n = 5$. As expected from the framework’s structure, higher values of ξ —indicating greater immediate experience recognition—lead to lower dispersion in the distribution of i_t . This is because more of the experience is realized immediately, leaving less volatility to accumulate in future periods. Notably, the case where $\xi = 0$ exactly replicates the distribution observed under the second approach with $n = 5$, which confirms the consistency of the framework.

Turning now to retirement benefits, Figure 4.11 presents the distribution of benefits in real terms for the hybrid approach for several cohorts using fixed values of $n = 25$ and $\xi = 0.50$. The overall trends are consistent with those observed in Figure 4.6 from the second approach. However, we observe slightly less dispersion in outcomes, attributable to the partial immediate recognition of experience in this design. This mechanism curtails the accumulation of large, unrecognized notional surpluses or deficits, yielding a narrower distribution of benefit outcomes.

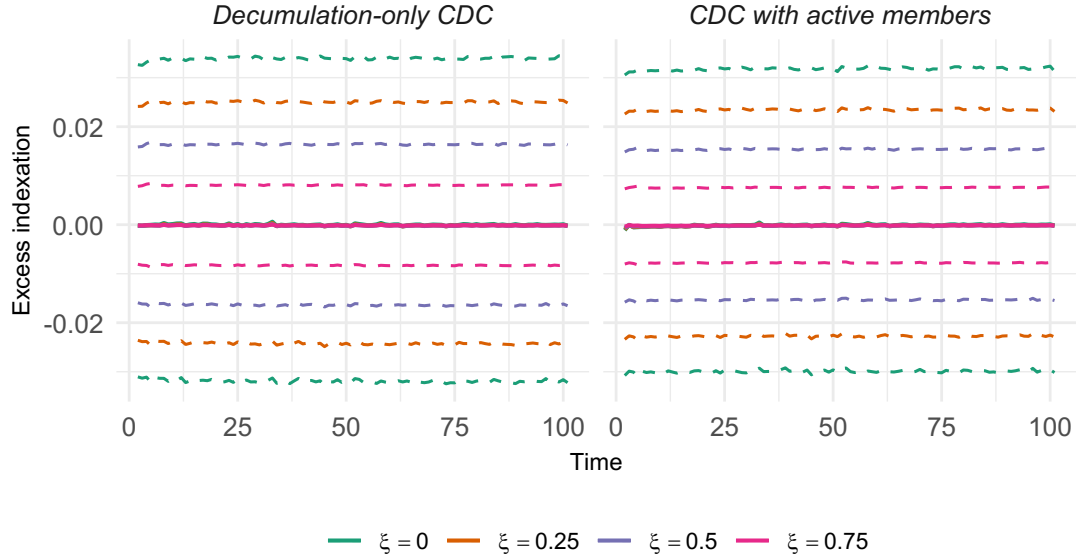


Figure 4.10: **Distribution of excess indexation i_t for the hybrid adjustment mechanism approach for varying ξ and a constant $n = 5$.**

Notes: This figure shows the distribution of excess indexation i_t for the hybrid adjustment mechanism approach with a fixed smoothing window $n = 5$ across various levels of immediate experience recognition ξ , with the left panel corresponding to the decumulation-only plan, and the right panel to the plan including active members. The solid lines represent the average, and the dashed lines are the 5th and 95th percentiles of the distribution.

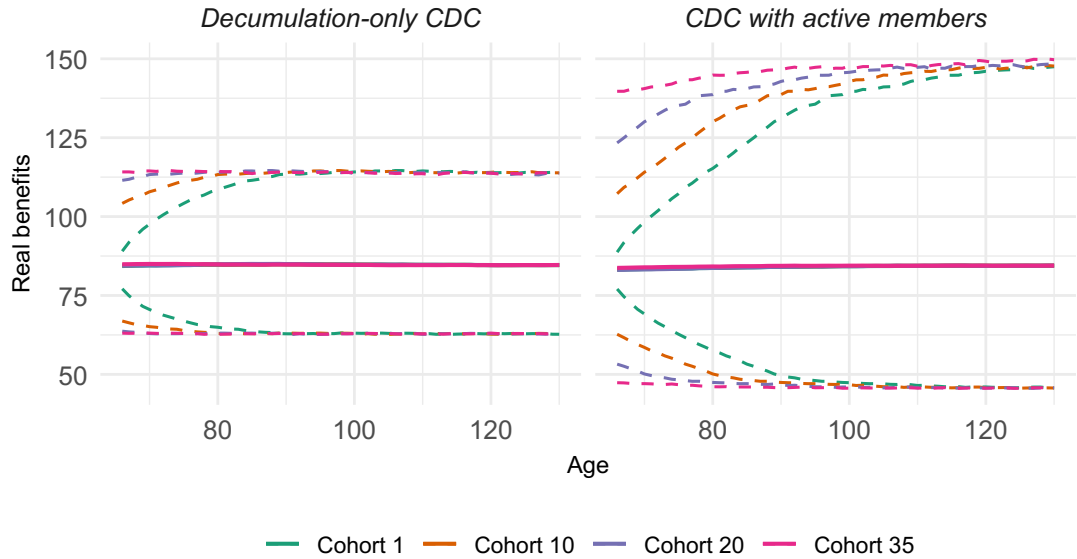


Figure 4.11: **Distribution of benefits in real terms for the hybrid adjustment mechanism approach for constant $n = 25$ and $\xi = 0.50$.**

Notes: This figure shows the benefits in real terms for the hybrid adjustment mechanism approach for $n = 25$ and $\xi = 0.5$, across various cohorts, with the left panel corresponding to the decumulation-only plan, and the right panel to the plan including active members. The solid lines represent the average, and the dashed lines are the 5th and 95th percentiles of the distribution.

Finally, we isolate the experience of a single cohort—Cohort 1 here—and evaluate the influence of varying n while keeping $\xi = 0.50$ fixed in Figure 4.12. As in the second approach, longer smoothing horizons result in more stable, gradual changes to benefits. However, they also increase intergenerational risk sharing, which amplifies the range of potential long-term outcomes. Conversely, a smaller n results in more immediate adjustments, leading to tighter distributions. Although this effect is much smaller in Figure 4.12 compared to the gradual recognition approach shown in Figure 4.7. This is mainly due to the immediate recognition of half of last year’s investment and mortality experience.

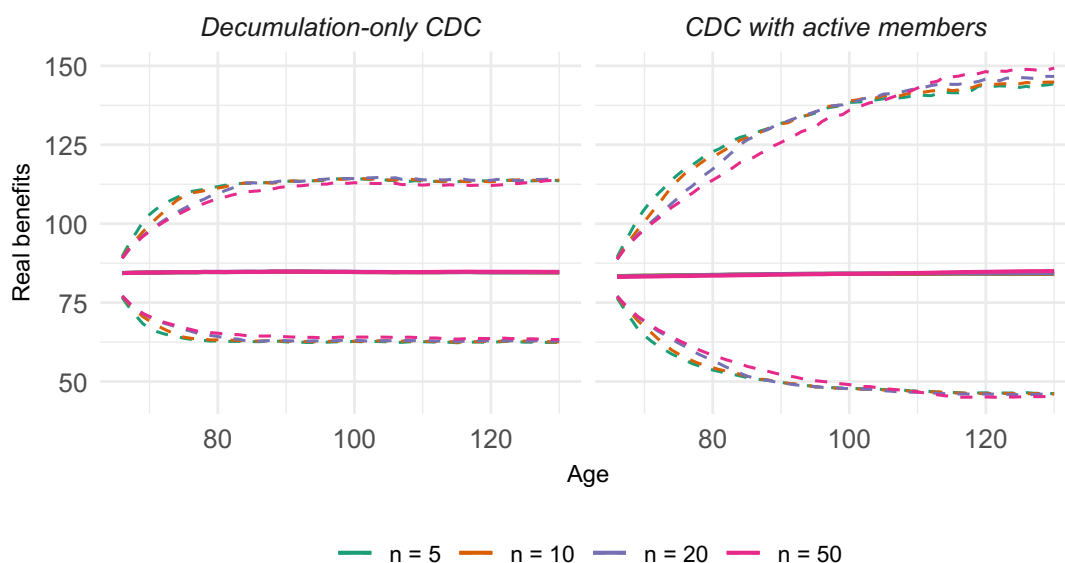


Figure 4.12: **Distribution of benefits in real terms for the hybrid adjustment mechanism approach for varying n and a constant $\xi = 0.50$.**

Notes: This figure shows benefits in real terms for Cohort 1 under different choices of n values while holding $\xi = 0.50$ constant, with the left panel corresponding to the decumulation-only plan, and the right panel to the plan including active members. The solid lines represent the average, and the dashed lines are the 5th and 95th percentiles of the distribution.

Chapter 5

Evaluation of plan design and membership welfare

The existing literature on CDC and target benefit-style retirement plans suggests that the selection of performance metrics should be guided by the need to balance two fundamental objectives: benefit adequacy and benefit security (see, e.g., Blommestein et al., 2009; Broeders and Chen, 2010; Sanders, 2016; Alonso-García et al., 2018, among others). Adequacy ensures that benefits provide sufficient income to meet retirees’ financial needs, while security addresses the reliability of these benefits over time. To ensure long-term sustainability, a well-designed pension scheme must perform effectively in both aspects.

Regulatory frameworks for CDC plans emphasize the concept of “benefit security,” yet its interpretation varies. For some stakeholders, security implies a high probability that actual benefits will meet or exceed the initially targeted amounts (see Donnelly, 2017). For others, it signifies stability, meaning that benefits should not fluctuate excessively from year to year (see Chen et al., 2023; Kabuche et al., 2025). Since both interpretations are relevant, a comprehensive set of performance metrics must capture both the likelihood of achieving target benefits and the stability of benefit payments over time.

Intergenerational equity is another crucial consideration in assessing pension plan performance. A well-designed plan should ensure that no cohort of members is unduly advantaged or disadvantaged by the structure of contributions and benefit adjustments. To evaluate this, at least one metric should enable meaningful comparisons between different generations of plan members, ensuring that risks and rewards are shared equitably over time.

While certain aspects of pension plan evaluation inherently involve complex calculations, the chosen metrics should remain interpretable and practical. Stakeholders, including regulators, policymakers, plan managers and trustees, and plan participants must be able to

understand and apply these metrics without extensive actuarial expertise. Clear and accessible performance measures enhance transparency and facilitate informed decision-making.

To thoroughly assess the effectiveness of the proposed design, we introduce a comprehensive set of performance metrics that jointly capture key aspects of benefit security, benefit stability, and intergenerational fairness. These metrics serve as a robust foundation for evaluating both the financial viability of the pension scheme and the distributional impacts across successive cohorts. We organize them into three categories: those evaluating benefit security, those quantifying benefit stability, and those measuring intergenerational equity—each highlighting the inherent trade-offs arising from different ends of the proposed design spectrum. Measures of benefit adequacy are intentionally omitted, as they are solely determined by the level of contributions or preexisting assets brought into the plan by retirees, both of which are assumed to be fixed in this analysis. By encompassing all critical dimensions relevant to the CDC pension plan evaluation, this multidimensional framework enables a well-rounded analysis of scheme outcomes. A detailed explanation of each metric is provided in the following sections. The evaluation criteria considered are:

1. **Benefit security**

- Distribution of realized internal rate of return (IRR).
- Distribution of serial disappointment.

2. **Benefit stability**

- Roughness of benefit payments.
- Increment ratio (IR) smoothness of benefit payments.

3. **Intergenerational equity and trade-offs**

- Certainty equivalent consumption (CEC) based on the constant relative risk aversion (CRRA) utility.
- Interquantile range (IQR) instability of the compound annual growth rate (CAGR).
- Quantile-inequity and median-inequity in CAGR.

5.1 **Distribution of the internal rate of return**

While the distribution of the adjustment factor $\alpha(t)$ and excess indexation i_t offers insight into benefit adjustments over time, it does not capture the full sequence of contributions and benefits received throughout a member’s lifetime. To address this limitation, we examine the IRR achieved by each cohort under various design parameter choices. The IRR is defined as the discount rate that equates the net present value (NPV) of all cash flows at the time

of plan entry to zero. It reflects the rate at which the present value of contributions made and benefits received—adjusted for survivorship—balance out over the member’s lifetime.

For the k^{th} cohort, represented by a hypothetical member living to age τ , the IRR is obtained by numerically solving for $\tilde{i}$ in the equation

$$0 = -A_k(0) + \sum_{t=R-x_k}^{\tau-x_k} B_k(t) {}_t p_{x_k} (1 + \tilde{i})^{-t} - \sum_{t=0}^{\tau} C_k(t) {}_t p_{x_k} (1 + \tilde{i})^{-t}.$$

By examining the distribution of IRRs, we can assess whether certain cohorts experience systematically higher or lower returns, thereby identifying potential implicit wealth transfers between generations. This also allows us to evaluate whether the plan delivers on its promises—namely, whether members achieve the target IRR set at plan inception—and to quantify the extent of variability resulting from different design choices and times of retirement.

Figure 5.1 shows the impact of the design parameter n and retirement timing on IRR outcomes. The contrast between the decumulation-only design and the inclusion of active members is especially noteworthy. Across all values of n , the decumulation-only design exhibits increasing IRR variance for successive cohorts compared to Cohort 1, which retires at inception. In contrast, the design incorporating active members exhibits decreasing IRR variance across cohorts.

This behaviour is driven by cohort exposure time. In the decumulation-only design, each cohort remains in the plan for the same duration. Hence, with longer smoothing windows, the likelihood of accumulating unrecognized surplus or deficit increases, which in turn amplifies the IRR dispersion for later cohorts. Conversely, smaller values of n lead to heightened volatility for early cohorts, as more experience is immediately recognized, resulting in less smoothing of outcomes. Higher values of n defer experience recognition, stabilizing benefits—an effect more pronounced for early cohorts.

In contrast, in the CDC design with active members, Cohort 1 enters at age 65, and each subsequent cohort spends one additional year in the plan compared to its predecessor. This extended exposure reduces the dispersion in realized IRRs across cohorts over time and allows IRRs to converge toward their asymptotic distribution. While differences across values of n remain evident for early cohorts—similar in shape to the decumulation-only design—the trend reverses over time. Moreover, the overall dispersion is lower in the plan with actives, due to the larger participant pool size.

Figure 5.2 presents similar findings for varying values of ξ and a fixed $n = 5$. As ξ decreases, the plan shifts away from immediate recognition (first approach) with $\xi = 1$ towards gradual recognition (second approach) with $\xi = 0$, deferring the recognition of all experience

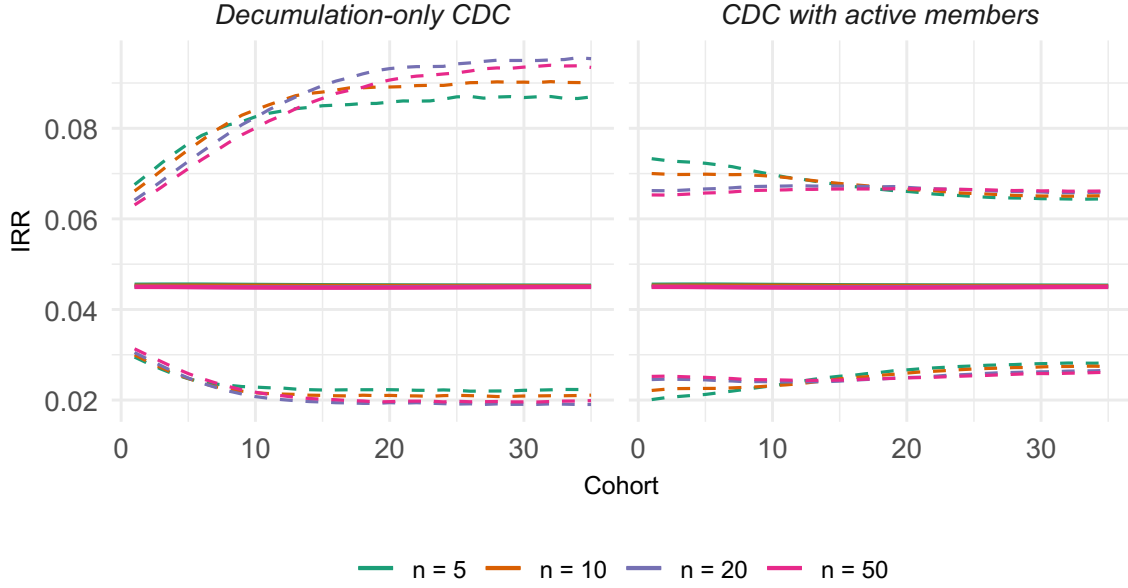


Figure 5.1: **Distribution of IRR across cohorts for varying n and a constant $\xi = 0$.**

Notes: This figure shows the distribution of IRRs for all cohorts and for different values of n , while holding $\xi = 0$ constant (gradual recognition of experience approach), with the left panel corresponding to the decumulation-only plan, and the right panel to the plan including active members. The solid lines represent the average, and the dashed lines are the 5th and 95th percentiles of the distribution.

uniformly over n years. This shift mirrors the effects observed when increasing n . For $\xi = 0$, adjustments for initial cohorts are muted, but the deferral of experience may result in greater accumulation of notional surplus or deficit, increasing IRR dispersion for later cohorts. Conversely, higher ξ values result in more immediate adjustments, producing greater volatility for early cohorts, but ultimately leading to tighter bounds around the long-term distribution of IRRs.

5.2 Distribution of serial disappointment

While the potential for reductions in benefits is an inherent feature of CDC and other risk-sharing plans, the frequency and duration of such reductions can strongly influence member perceptions and overall plan sustainability. If benefit reductions occur too frequently or the plan persistently lags behind the target, members may become dissatisfied and consider exiting. Generally, members expect their benefits to at least remain on target, or to exceed the target over time due to positive net indexation. Any deviation from this expectation—particularly in the form of reductions—can trigger dissatisfaction, which we define as member disappointment.

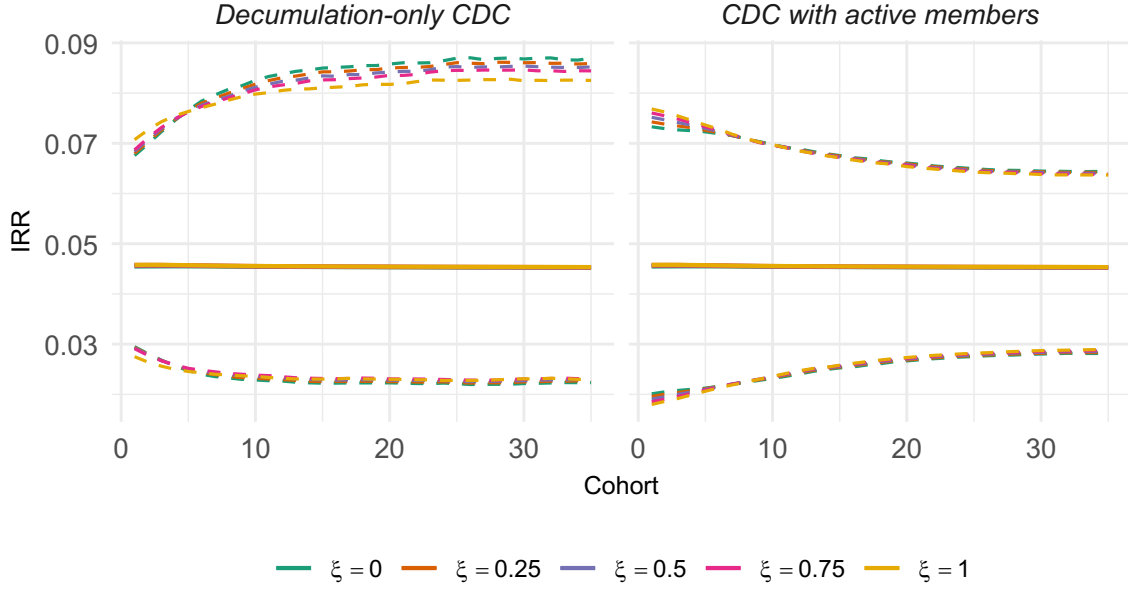


Figure 5.2: **Distribution of IRR across cohorts for varying ξ and a constant $n = 5$.**

Notes: This figure shows the distribution of IRRs for all cohorts under different values of ξ with $n = 5$ fixed, thereby comparing all approaches, with the left panel corresponding to the decumulation-only plan, and the right panel to the plan including active members. The solid lines represent the average, and the dashed lines are the 5th and 95th percentiles of the distribution.

Member disappointment is conceptually linked to disappointment aversion, as introduced by Gul (1991). This measure is used by Donnelly (2017) in the context of risk-sharing plans, where the disappointment is considered in terms of consecutive years of benefit decreases. Adapting this metric to our context, we propose quantifying the disappointment experienced by retired members by evaluating the probability of benefit shortfalls relative to the target. Specifically, we focus on serial disappointment: the likelihood of experiencing consecutive years of benefit payments below the target level. Members are more likely to perceive sustained underperformance as unacceptable, making this a relevant and informative measure of perceived satisfaction.

To quantify this, we count the occurrences of exactly two consecutive years in which the actual benefit paid falls below the target, across all simulations. This means if there were a run of four consecutive years in which the actual benefit paid was less than the target benefit amount, it would be counted as three instances of two-year disappointment. We then extend this analysis to runs of three, four, five or more consecutive years, thereby constructing a probability distribution for serial disappointment across different time horizons. This allows us to explicitly measure the risk that a member remains disappointed for an extended period.

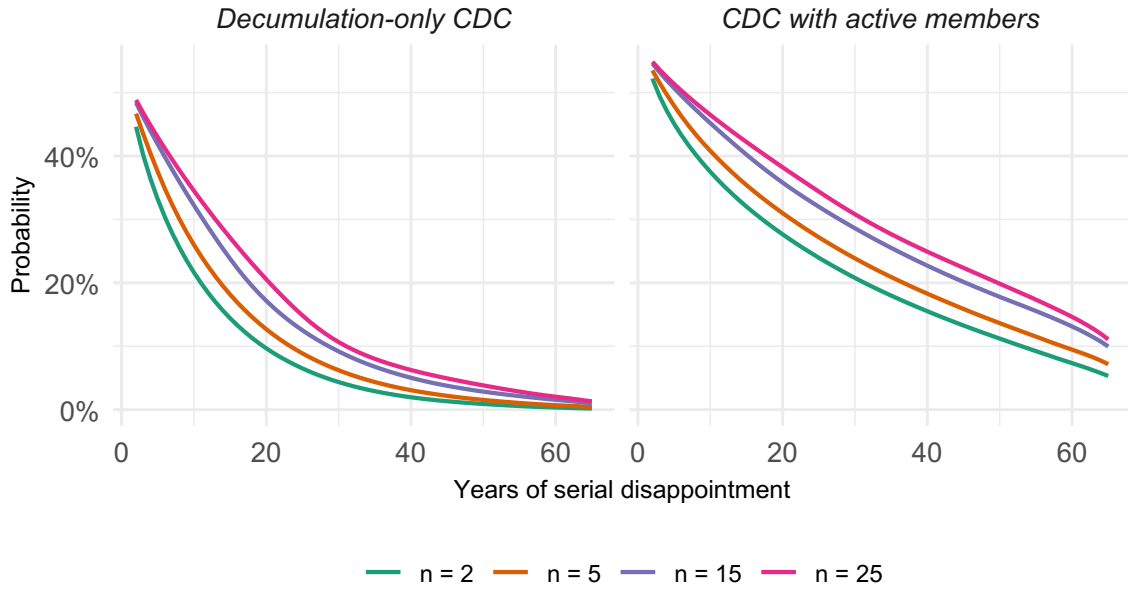


Figure 5.3: **Probability of serial disappointment for varying n and a constant $\xi = 0$.**

Notes: This figure shows the probability of serial disappointment (i.e., actual benefits falling below the target for a certain number of years consecutively) under different values of n , while holding $\xi = 0$ constant (gradual recognition of experience approach), with the left panel corresponding to the decumulation-only plan, and the right panel to the plan including active members.

Figure 5.3 illustrates the impact of the smoothing window on serial disappointment. As the size of the smoothing window n increases, the probability of experiencing prolonged underperformance also rises. This is expected: longer smoothing delays the recognition of notional deficits, accumulating over time and suppressing benefits relative to target levels for extended periods. We would observe the same pattern if we were looking at the probability of consecutive years of realized benefits being higher than the target level (i.e., serial delight), due to the accrual of notional surplus. The presence of active members exacerbates this effect, due to additional dynamics introduced by contributions, risk absorption by active members, and a larger pool size, and we consequently observe higher probabilities of serial disappointment in the design with active members compared to the decumulation-only design.

Similarly, the impact of ξ on serial disappointment is shown in Figure 5.4. As ξ increases from 0 to 1—shifting from gradual recognition to immediate and full recognition—the probability of serial disappointment decreases. This reflects more immediate recognition of investment and mortality experience, resulting in faster corrective adjustments and reduced persistence of underperformance. Although the difference between $\xi = 0$ and $\xi = 0.25$ appears non-existent for the example shown at $n = 5$, it becomes more pronounced for larger values of n , where the deferral of experience has a more substantial effect on benefit trajectories.

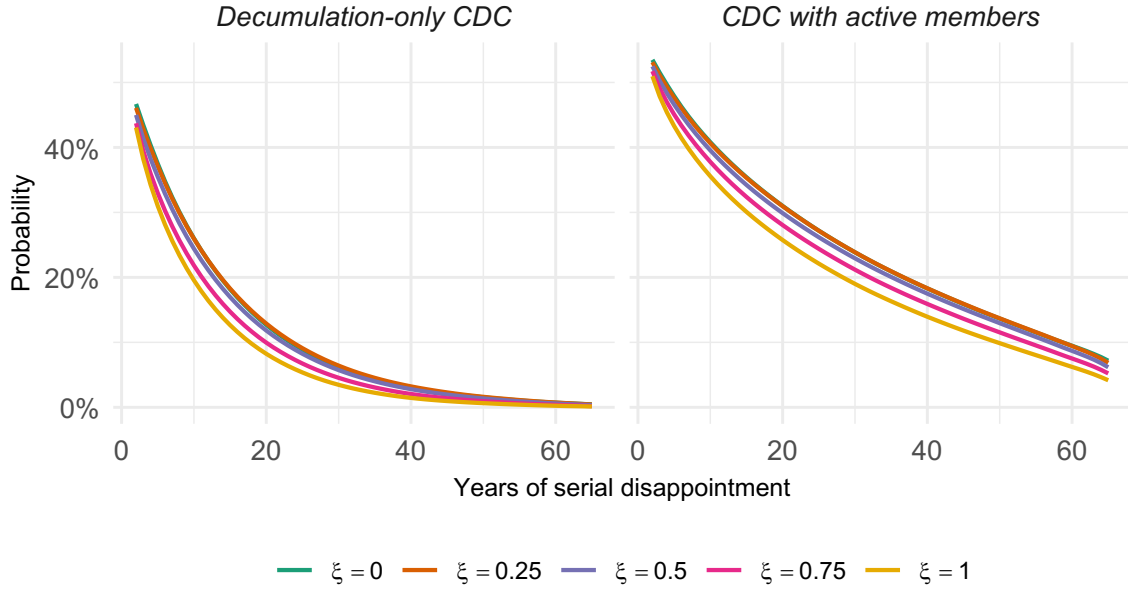


Figure 5.4: **Probability of serial disappointment for varying ξ and a constant $n = 5$.**

Notes: This figure shows the probability of serial disappointment (i.e., actual benefits falling below the target for a certain number of years consecutively) under different values of ξ , with $n = 5$ fixed, thereby comparing all approaches, with the left panel corresponding to the decumulation-only plan, and the right panel to the plan including active members.

5.3 Roughness of benefit payments

Previously, we noted that there are two complementary interpretations of benefit security: the first focused on the level of benefits—namely, whether benefit amounts exceed or at least remain on par with the target—and the second, developed here, is concerned about the stability of benefit payments over time. While the former relates to security, the latter pertains to the smoothness and predictability of the benefit stream, which can influence member satisfaction and perceptions of fairness. In this sense, stability refers to situations where the magnitude and direction of changes in benefit amounts are relatively small.

We introduce two metrics to quantify benefit stability. The first, developed in this section, captures the roughness of the benefit payment stream—drawing on a concept from calculus—and the second, based on increment ratios, is introduced in the following section.

In calculus, smoothness (and by extension, roughness, i.e., lack of smoothness) is closely associated with the differentiability of a function, which is formally measured as the number of continuous derivatives it possesses over its domain. In the context of pension benefit payments, we define roughness as the rate of change in the rate of change of benefit levels, effectively capturing how frequently and abruptly the direction of benefit adjustments varies over time. This is also known as the curvature of a function, and it measures how fast

a curve is changing direction at a given point. Lower roughness implies slower directional changes in benefit updates, corresponding to greater benefit stability and smoother transitions in payment levels. From the perspective of both plan stakeholders and members, such smooth adjustments are generally more desirable, as they reduce volatility and enhance predictability in retirement income.

Since our benefit payments are modelled in discrete time, we employ the squared second-order difference of benefits in real terms to construct a discrete analogue of curvature. This measure captures the extent of directional changes in benefit adjustments and serves as a proxy for the roughness of the benefit trajectory. To summarize the roughness of a given plan design, we compute the average of this quantity across all cohorts, all time periods within the retirement span, and all simulation paths, thereby yielding a single scalar value. Squaring the second difference isolates the magnitude of fluctuations, rather than their direction. The squared second-order difference of benefits in real terms for the k^{th} cohort at time t is given by

$$\begin{aligned} \left(\Delta^2 B'_k(t)\right)^2 &= \left[(B'_k(t+2) - B'_k(t+1)) - (B'_k(t+1) - B'_k(t))\right]^2 \\ &= \left[B'_k(t+2) - 2B'_k(t+1) + B'_k(t)\right]^2, \end{aligned}$$

where $B'_k(t)$ denotes the benefit in real terms received at time t by cohort k , obtained by discounting the nominal benefit $B_k(t)$ using the target indexation rate i over t years. The roughness metric for a given plan design is then defined as

$$\text{Roughness} = \mathbb{E} \left[\frac{1}{35} \sum_{k=1}^{35} \frac{1}{\tau - R - 1} \sum_{t=R-x_k}^{\tau-x_k-2} \left(\Delta^2 B'_k(t)\right)^2 \right],$$

where the expectation is taken over all simulation paths for all 35 cohorts under consideration.

Table 5.1 summarizes the roughness values for different combinations of design parameters n and ξ in the CDC plan with active members. As expected, we observe a consistent decline in roughness with increasing smoothing window length n , for any fixed value of ξ . Similarly, roughness decreases as ξ is reduced—that is, as the plan design moves away from immediate recognition of experience to gradual recognition. Importantly, the table reveals that comparable roughness levels can be achieved through different combinations of n and ξ , providing regulators, policymakers, plan managers, and trustees with flexibility to balance benefit stability against other competing objectives, such as minimizing member disappointment.

Table 5.2 presents the corresponding results for the decumulation-only plan. The patterns are largely similar to the plan with actives: increasing n and decreasing ξ reduce benefit roughness. A few minor deviations are observed at high values of ξ (notably, $\xi = 0.75$

Table 5.1: Roughness for the CDC plan with active members.

Panel A: Full and immediate recognition of experience										
$\xi = 1.00$										
91.71										
Panel B: Gradual recognition of experience over the next n years										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.00$	25.84	12.58	7.77	5.45	2.15	1.48	1.25	1.15	0.85	0.67
Panel C: Hybrid adjustment mechanism										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.25$	29.52	19.25	15.29	13.22	9.92	9.18	8.94	8.86	8.62	8.48
$\xi = 0.50$	41.89	34.84	31.96	30.37	27.67	27.05	26.86	26.82	26.57	26.46
$\xi = 0.75$	62.69	59.07	57.52	56.61	55.01	54.64	54.53	54.50	54.29	54.25

Notes: This table reports the average roughness of real benefit payments across all cohorts and over the full projection horizon for the CDC design with active members.

Table 5.2: Roughness for the decumulation-only CDC plan.

Panel A: Full and immediate recognition of experience										
$\xi = 1.00$										
89.80										
Panel B: Gradual recognition of experience over the next n years										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.00$	27.25	13.52	8.50	6.07	2.58	1.88	1.64	1.53	1.11	0.90
Panel C: Hybrid adjustment mechanism										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.25$	31.02	20.50	16.31	14.12	10.68	9.95	9.72	9.64	9.35	9.24
$\xi = 0.50$	44.07	36.88	33.79	32.09	29.26	28.66	28.49	28.45	28.25	28.25
$\xi = 0.75$	66.25	62.55	60.85	59.87	58.18	57.83	57.75	57.76	57.67	57.76

Notes: See the caption of Table 5.1.

beyond $n = 20$), which are likely attributable to the smaller size of the retiree-only population, shorter benefit durations, and limited ability to smooth over very long time horizons. Furthermore, when comparing the two designs, we find that the plan with active members generally exhibits lower roughness for equivalent parameter choices—except for the full and immediate recognition approach, where smoothing is absent. This difference arises from the larger risk-sharing pool in the active-member design, which enhances stability by absorbing shocks more effectively and yielding more predictable mortality experience.

5.4 Increment ratio smoothness

Building on the calculus-inspired roughness metric introduced in the previous section, we now turn to a geometric perspective of benefit payment stability. Specifically, we adopt a measure originally proposed by Bardet and Surgailis (2011), which evaluates the smoothness of a stochastic process via the frequency and structure of sign changes in local increments—referred to as increment ratio (IR) smoothness.

To adapt this framework to our context, let $B'_k(t)$ denote the time- t benefit in real terms (discounted to time $t = 0$ at target rate of benefit increase i) received by the k^{th} cohort. With the convention that $\frac{0}{0} \equiv 1$, the first-order IR smoothness of k^{th} cohort is defined as follows:

$$\begin{aligned} \text{IR smoothness}_k &= \frac{1}{99} \sum_{j=0}^{98} \frac{|B'_k(j+1) - B'_k(j) + B'_k(j+2) - B'_k(j+1)|}{|B'_k(j+1) - B'_k(j)| + |B'_k(j+2) - B'_k(j+1)|} \\ &= \frac{1}{99} \sum_{j=0}^{98} \frac{|B'_k(j+2) - B'_k(j)|}{|B'_k(j+2) - B'_k(j+1)| + |B'_k(j+1) - B'_k(j)|}. \end{aligned}$$

which is the first-order IR smoothness for the k^{th} cohort over the 100 year simulation time horizon. This expression effectively captures the directional consistency of adjacent increments in the benefit stream. The numerator is always less than or equal to the denominator due to the triangle inequality, ensuring that the IR smoothness lies within the closed interval $[0, 1]$. Averaging this value across all 35 cohorts under consideration to get a scalar representing IR smoothness for the whole plan design yields:

$$\text{IR smoothness} = \mathbb{E} \left[\frac{1}{35} \left(\sum_{k=1}^{35} \text{IR smoothness}_k \right) \right]$$

Intuitively, IR smoothness attains values close to 1 when benefit adjustments are unidirectional or monotonic over time, reflecting a high degree of temporal consistency in benefit changes. Conversely, values near 0 indicate frequent reversals in the direction of year-on-year changes, corresponding to high volatility or *roughness* in the payment path. Bardet

and Surgailis (2011) establish that IR smoothness converges to 1 in the limit for sufficiently smooth functions.

Tables 5.3 and 5.4 report the IR smoothness across different parameter configurations for the CDC design with and without active members, respectively. The empirical patterns align with expectations: IR smoothness increases with the smoothing window length n , and also increases as the immediate experience recognition rate ξ decreases—moving from full recognition with $\xi = 1$ to gradual recognition with $\xi = 0$.

Consistent with findings from the curvature-based metric in the previous section, the plan with active members exhibits higher IR smoothness across most combinations of n and ξ , attributable to the larger and more stable risk-sharing pool. The exception remains the case of full and immediate recognition, where no smoothing is applied in either design, leading to comparably low smoothness metrics.

We advocate IR smoothness as a preferred metric of benefit stability over the curvature-based roughness measure, primarily due to its bounded and interpretable scale between 0 and 1. It facilitates intuitive communication of stability outcomes to plan participants and stakeholders: a value of 1 implies a perfectly smooth benefit trajectory with no directional reversals, while a value near 0 suggests highly erratic, oscillating payment patterns.

5.5 Certainty equivalent consumption based on constant relative risk aversion utility

To analyze intergenerational risk-sharing and the trade-offs inherent in CDC plans from a member welfare perspective—specifically the balance between benefit level and security—we adopt a standard framework based on expected utility. In particular, we evaluate plan designs within a CRRA utility framework, which allows us to quantify how individuals with specific risk and time preferences value a given income stream. These valuations are then expressed in terms of CEC, defined as the level of guaranteed income that renders the individual indifferent between participation in the CDC plan and receiving a fixed life annuity of that amount.

Under the CRRA utility framework, the expected lifetime utility for an individual at inception is given by

$$U = \mathbb{E} \left[\sum_{t=R-x_k}^{\tau-x_k} \frac{B(t)^{1-\gamma}}{1-\gamma} {}_t p_{x_k} e^{-\delta t} \right],$$

where $B(t)$ denotes the time- t benefit, $\gamma \in [0, 0) \cup (1, \infty]$ is the coefficient of relative risk aversion, and δ is the subjective discount rate. In line with moderate risk preference assumptions frequently found in the literature (see Hanewald et al., 2013; Kabuche et al., 2025), we set $\gamma = 2$ and assume a subjective discount factor of $e^{-\delta} = 0.98$.

Table 5.3: IR smoothness for the CDC plan with active members.

Panel A: Full and immediate recognition of experience										
$\xi = 1.00$										
0.723										
Panel B: Gradual recognition of experience over the next n years										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.00$	0.824	0.858	0.876	0.887	0.910	0.916	0.918	0.918	0.919	0.923
Panel C: Hybrid adjustment mechanism										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.25$	0.812	0.825	0.828	0.827	0.817	0.809	0.803	0.799	0.789	0.787
$\xi = 0.50$	0.785	0.781	0.776	0.771	0.757	0.750	0.746	0.744	0.741	0.740
$\xi = 0.75$	0.753	0.746	0.742	0.739	0.732	0.729	0.728	0.728	0.727	0.727

Notes: This table shows the mean of IR smoothness across all cohorts and time horizons in consideration for the CDC plan with active members.

Table 5.4: IR smoothness for the decumulation-only CDC plan.

Panel A: Full and immediate recognition of experience										
$\xi = 1.00$										
0.730										
Panel B: Gradual recognition of experience over the next n years										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.00$	0.814	0.850	0.868	0.880	0.903	0.909	0.909	0.908	0.909	0.913
Panel C: Hybrid adjustment mechanism										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.25$	0.802	0.814	0.816	0.816	0.806	0.798	0.792	0.787	0.777	0.775
$\xi = 0.50$	0.773	0.768	0.762	0.757	0.742	0.736	0.732	0.731	0.728	0.727
$\xi = 0.75$	0.740	0.732	0.728	0.725	0.718	0.715	0.714	0.714	0.714	0.713

Notes: See the caption of Table 5.3.

After computing expected utility across all simulated benefit paths, we solve for the CEC by finding the level benefit that equates this utility to that from a constant stream:

$$U = \sum_{t=R-x_k}^{\tau-x_k} \frac{(\text{CEC})^{1-\gamma}}{1-\gamma} {}_t p_{x_k} e^{-\delta t}.$$

Figure 5.5 shows the resulting CECs across various smoothing window lengths n and a fixed value of $\xi = 0$ (i.e., gradual recognition of experience approach). We observe a U-shaped relationship between the smoothing parameter n and the CEC. For shorter smoothing horizons, the CEC initially declines as smoothing is introduced—this decline reflects the cost of smoothing, where plan members trade a higher but more volatile benefit stream for lower but more stable payments. However, as the length of the smoothing window increases, CEC recovers and eventually surpasses initial levels. This suggests that over sufficiently long horizons, the benefits of stability and deferred experience recognition outweigh the initial loss in benefit level.

A notable outlier is Cohort 1, which sees a monotonic increase in utility with longer smoothing windows. This arises because there is no possibility of preexisting notional deficits at their entry, allowing them to benefit from smoothing without incurring any historical negative experience. In contrast, later cohorts may inherit adverse smoothing effects.

The same qualitative trends persist across other values of ξ : the least smoothed design delivers the highest CEC, and it decreases as ξ decreases. The plan with active members shows similar trends compared to the plan without active members, although the dispersion in CECs between cohorts is more pronounced. This is because successive cohorts join the plan earlier in life, face longer time horizons, and bear greater uncertainty over whether they will ultimately receive any benefits, thereby reducing their initial CEC.

5.6 Interquantile range instability

A number of quantile-based measures of intergenerational equity were introduced by Donnelly (2017): she uses the consistency of plan performance between cohorts to measure the inequity that may arise from the intergenerational risk-sharing features of a CDC plan. The first such measure is IQR instability. It captures the consistency of dispersion in outcomes across different cohorts. In our analysis, we modify Donnelly’s (2017) IQR instability metric by using the CAGR as the relevant measure of plan performance, and we examine various quantiles of the achieved CAGR across simulations for each cohort. Rather than focusing on averages, IQR instability focuses on the dispersion of outcomes by measuring the spread between selected upper and lower quantiles. Specifically, for each cohort k , we define the

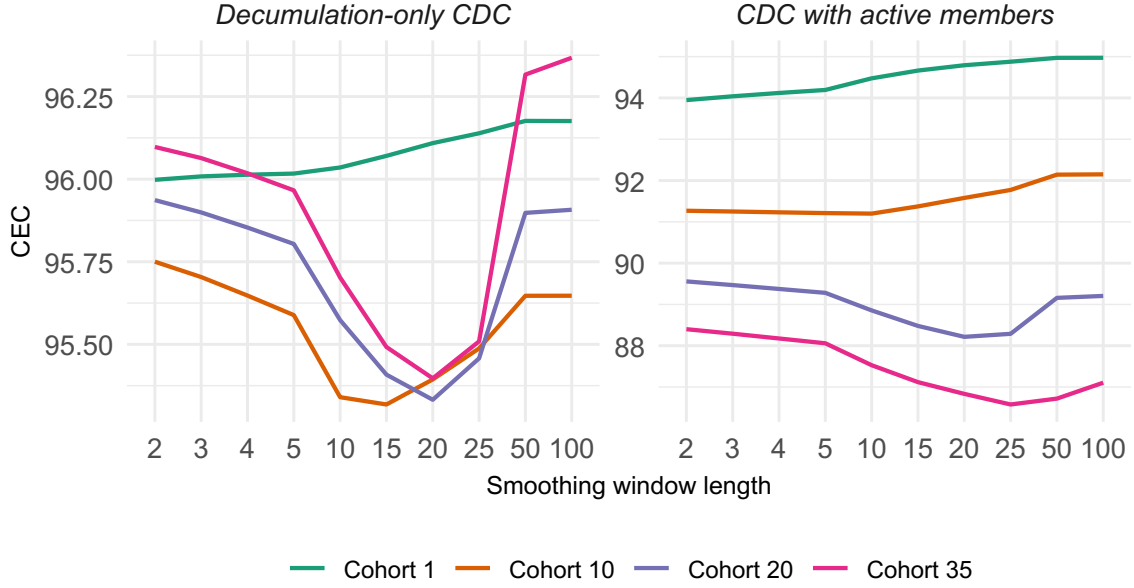


Figure 5.5: **Certainty equivalent consumption under CRRA utility.**

Notes: This figure shows the CEC of expected utility under CRRA utility with $\gamma = 2$. The value of $\xi = 0$ is held constant across all cohorts, with the left panel corresponding to the decumulation-only plan, and the right panel to the plan including active members.

CAGR as

$$\text{CAGR}_k = \frac{1}{\tau - R} \log \left(\frac{B_k(\tau - x_k)}{B_k(R - x_k)} \right).$$

leading to IQR of CAGR for the k^{th} cohort as

$$\begin{aligned} \text{IQR}_k &= 95^{\text{th}} \text{ percentile of CAGR for } k^{\text{th}} \text{ cohort} \\ &\quad - 5^{\text{th}} \text{ percentile of CAGR for } k^{\text{th}} \text{ cohort.} \end{aligned}$$

A lower IQR of the CAGR implies reduced dispersion in the realized growth rates of benefit payments for a given cohort. This indicates greater stability in benefit streams and less variability in the outcomes experienced by cohort members year-over-year. Conversely, a higher IQR of CAGR suggests greater dispersion in the realized growth rates, reflecting a more volatile benefit trajectory. In such cases, the cohort in question would have encountered more frequent and substantial adjustments to their benefits over time.

Based on this, we define IQR instability as the difference between the maximum and minimum cohort-specific IQR values:

$$\text{IQR instability} = \max_{k \in \{1, 2, \dots, 35\}} \text{IQR}_k - \min_{k \in \{1, 2, \dots, 35\}} \text{IQR}_k.$$

An equitable plan design should ideally maintain a stable IQR across generations. Table 5.5 presents the IQR instability across a variety of design configurations for the CDC plan with active members. Across different combinations of smoothing window length n and immediate experience recognition rate ξ , the IQR instability remains relatively low, typically within the range of 0.3% to 0.6%. This indicates that the spread of potential CAGR outcomes between cohorts is narrowly confined, and no cohort is subject to significantly greater benefit volatility than others.

Here, we observe an inverted U-shaped relationship between IQR instability and the smoothing window length n . Initially, as n increases, IQR instability also increases. This is intuitive, as longer smoothing horizons imply slower recognition of plan experience, allowing larger notional surpluses or deficits to accumulate—some of which may outlive a given cohort’s participation in the plan. This leads to intergenerational wealth transfers and a wider range of outcomes across cohorts. However, for very large n , this trend reverses. In the limit as $n \rightarrow \infty$, IQR instability may approach or even improve upon that observed under the full and immediate recognition approach with $\xi = 1$, which applies no smoothing at all.

We also find that increasing ξ —thereby recognizing a greater portion of the experience immediately—tends to reduce IQR instability. This aligns with the interpretation that instability is largely driven by the deferral and subsequent unwinding of built-up notional deficits and surpluses. When $\xi = 0$, experience is fully deferred, which may lead to higher intergenerational dispersion in benefit paths due to these accumulations.

We see similar trends continue in IQR instability for the decumulation-only design. Table 5.6 demonstrates that IQR instability follows the same qualitative pattern as in the plan with active members; however, the level of instability is noticeably lower across the board. This can be attributed to the fact that, for decumulation-only plans, each cohort spends an identical amount of time in the plan. In contrast, in the active-members design, successive cohorts spend progressively longer durations in the plan, which allows for more pronounced intergenerational effects and hence greater potential for instability, even for the same values of n and ξ .

In this context, it is especially noteworthy that, for the full and immediate recognition approach under the decumulation-only design, the IQR instability is observed to be approximately 0.02%, which is very close to the theoretical value of 0. This provides a useful validation of the underlying intuition: with no smoothing and no deferral of experience recognition, there are minimal intergenerational effects, and hence, virtually no variation in outcome dispersion between cohorts. Notably, all values for IQR instability remain below 0.6%, indicating that, regardless of the chosen design parameters n and ξ , and irrespective of whether the plan includes active members or operates in a decumulation-only setting, no cohort experiences materially greater dispersion in outcomes relative to others.

Table 5.5: IQR instability for the CDC plan with active members.

Panel A: Full and immediate recognition of experience										
$\xi = 1.00$										
0.4155										
Panel B: Gradual recognition of experience over the next n years										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.00$	0.4288	0.4387	0.4470	0.4578	0.4996	0.5262	0.5471	0.5679	0.4900	0.4680
Panel C: Hybrid adjustment mechanism										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.25$	0.4240	0.4319	0.4334	0.4372	0.4799	0.4892	0.4979	0.5210	0.5050	0.4061
$\xi = 0.50$	0.4217	0.4209	0.4289	0.4299	0.4467	0.4476	0.4511	0.4682	0.4959	0.3402
$\xi = 0.75$	0.4214	0.4183	0.4208	0.4248	0.4241	0.4206	0.4167	0.4334	0.4478	0.3077

Notes: This table shows the IQR instability in percentage of realized CAGR for various combinations of design parameters n and ξ in the CDC plan with active members.

Table 5.6: IQR instability for the decumulation-only CDC plan.

Panel A: Full and immediate recognition of experience										
$\xi = 1.00$										
0.0257										
Panel B: Gradual recognition of experience over the next n years										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.00$	0.3924	0.3998	0.4020	0.4033	0.4213	0.4342	0.4413	0.4487	0.2217	0.3724
Panel C: Hybrid adjustment mechanism										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.25$	0.3899	0.3931	0.3886	0.3908	0.3906	0.3917	0.4006	0.4099	0.2780	0.3544
$\xi = 0.50$	0.3874	0.3917	0.3850	0.3851	0.3762	0.3669	0.3734	0.3731	0.3607	0.3671
$\xi = 0.75$	0.3905	0.3905	0.3906	0.3878	0.3770	0.3736	0.3691	0.3703	0.4107	0.3663

Notes: See the caption of Table 5.5.

5.7 Quantile inequity

Quantile inequity, introduced by Donnelly (2017), is a measure intended to capture the overall disparity in outcomes across all cohorts. While IQR instability focused on the consistency of outcome dispersion between cohorts, quantile inequity highlights the spread between the best and worst outcomes observed across all generations. A lower value of quantile inequity indicates that the range of possible outcomes is relatively uniform across generations—suggesting that earlier generations are not unduly favoured at the expense of later ones, or vice versa.

In line with our use of the CAGR as the summary metric for cohort-level outcomes, we define quantile inequity as

$$\begin{aligned} \text{Quantile inequity} = & \max_{k \in \{1, 2, \dots, 35\}} 95^{\text{th}} \text{ percentile of CAGR for the } k^{\text{th}} \text{ cohort} \\ & - \min_{k \in \{1, 2, \dots, 35\}} 5^{\text{th}} \text{ percentile of CAGR for the } k^{\text{th}} \text{ cohort.} \end{aligned}$$

Table 5.7 reports the values of quantile inequity for the CDC plan with active members, across various combinations of smoothing window length n and immediate experience recognition rate ξ . We observe trends similar to those seen for IQR instability in Table 5.5. In particular, there is a clear U-shaped relationship between quantile inequity and the smoothing window n : as n increases, quantile inequity initially increases—likely due to the accumulation of long-term notional surpluses or deficits—but subsequently declines for very long smoothing horizons. Similarly, increasing the value of ξ , thereby recognizing more of the experience immediately, leads to lower quantile inequity.

These trends persist for the decumulation-only plan design, as seen in Table 5.8. However, consistent with our earlier findings for IQR instability, the levels of quantile inequity are significantly lower across all parameter values. This again reflects the same plan participation durations across cohorts in the decumulation-only setting, reducing the scope for intergenerational transfer effects.

An alternative, narrower measure of cross-cohort disparity focuses on median outcomes rather than tail extremes. Rather than evaluating the spread between best- and worst-case scenarios, we examine the range of median outcomes across cohorts. We adapt Donnelly’s (2017) median inequity to our context:

$$\begin{aligned} \text{Median inequity} = & \max_{k \in \{1, 2, \dots, 35\}} 50^{\text{th}} \text{ percentile of CAGR for } k^{\text{th}} \text{ cohort} \\ & - \min_{k \in \{1, 2, \dots, 35\}} 50^{\text{th}} \text{ percentile of CAGR for } k^{\text{th}} \text{ cohort.} \end{aligned}$$

Table 5.7: Quantile inequity for the CDC plan with active members.

Panel A: Full and immediate recognition of experience										
$\xi = 1.00$										
2.1572										
Panel B: Gradual recognition of experience over the next n years										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.00$	2.1776	2.1987	2.2219	2.2415	2.3310	2.4012	2.4544	2.5009	2.4554	2.3881
Panel C: Hybrid adjustment mechanism										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.25$	2.1671	2.1802	2.1918	2.2036	2.2687	2.3085	2.3380	2.3778	2.4015	2.2553
$\xi = 0.50$	2.1644	2.1701	2.1740	2.1801	2.2081	2.2292	2.2416	2.2685	2.3006	2.1282
$\xi = 0.75$	2.1605	2.1617	2.1616	2.1650	2.1719	2.1692	2.1726	2.1871	2.2177	2.0502

Notes: This table shows the quantile inequity in percentage of realized CAGR for various combinations of design parameters n and ξ in the CDC plan with active members.

Table 5.8: Quantile inequity for the decumulation-only CDC plan.

Panel A: Full and immediate recognition of experience										
$\xi = 1.00$										
1.0129										
Panel B: Gradual recognition of experience over the next n years										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.00$	1.3258	1.3414	1.3556	1.3711	1.4441	1.4932	1.5257	1.5536	1.2681	1.3740
Panel C: Hybrid adjustment mechanism										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.25$	1.3194	1.3246	1.3276	1.3343	1.3564	1.3728	1.3908	1.4046	1.2525	1.2785
$\xi = 0.50$	1.3195	1.3128	1.3116	1.3134	1.3034	1.2965	1.3007	1.2987	1.2746	1.2442
$\xi = 0.75$	1.3201	1.3151	1.3096	1.3046	1.2889	1.2809	1.2754	1.2698	1.2968	1.2460

Notes: See the caption of Table 5.7.

Table 5.9: Median inequity for the CDC plan with active members.

Panel A: Full and immediate recognition of experience										
$\xi = 1.00$										
0.0711										
Panel B: Gradual recognition of experience over the next n years										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.00$	0.0691	0.0702	0.0748	0.0723	0.0792	0.0870	0.0971	0.1048	0.0990	0.0984
Panel C: Hybrid adjustment mechanism										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.25$	0.0675	0.0692	0.0706	0.0713	0.0776	0.0820	0.0862	0.0969	0.0922	0.0811
$\xi = 0.50$	0.0707	0.0694	0.0687	0.0707	0.0745	0.0763	0.0790	0.0858	0.0899	0.0651
$\xi = 0.75$	0.0708	0.0704	0.0698	0.0698	0.0681	0.0698	0.0724	0.0750	0.0784	0.0530

Notes: This table shows the median inequity in percentage of realized CAGR for various combinations of design parameters n and ξ in the CDC plan with active members.

Table 5.10: Median inequity for the decumulation-only CDC plan.

Panel A: Full and immediate recognition of experience										
$\xi = 1.00$										
0.0058										
Panel B: Gradual recognition of experience over the next n years										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.00$	0.0156	0.0144	0.0141	0.0133	0.0134	0.0178	0.0127	0.0154	0.0119	0.0179
Panel C: Hybrid adjustment mechanism										
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 25$	$n = 50$	$n = 100$
$\xi = 0.25$	0.0156	0.0149	0.0124	0.0139	0.0106	0.0119	0.0079	0.0080	0.0119	0.0154
$\xi = 0.50$	0.0150	0.0151	0.0138	0.0125	0.0105	0.0092	0.0087	0.0088	0.0115	0.0149
$\xi = 0.75$	0.0157	0.0154	0.0142	0.0126	0.0109	0.0114	0.0114	0.0116	0.0125	0.0199

Notes: See the caption of Table 5.9.

This measure captures the generational dispersion in outcomes under a typical experience scenario. Tables 5.9 and 5.10 report the median inequity for CDC plans with and without active members, respectively. Again, we observe patterns similar to those found for quantile inequity. Notably, the decumulation-only plan exhibits substantially lower levels of median inequity, particularly under the full and immediate recognition approach when $\xi = 1$, where the reported value is just 0.006%. This is remarkably close to the theoretical value of zero, reflecting that, in this configuration, there is no smoothing and all cohorts spend the same length of time in the plan, thereby eliminating the mechanism for implicit intergenerational transfers.

Collectively, these three metrics—IQR instability, quantile inequity, and median inequity—demonstrate that under both plan types (with and without active members), and across a wide range of parameter choices for n and ξ , the CDC designs exhibit a high degree of consistency and fairness. Not only does the pricing approach ensure fairness from an *ex-ante* perspective (by pricing individuals solely based on future experience adjustments), but the observed low levels of *ex-post* inequity—particularly in median outcomes—suggest that the realized outcomes remain equitable across generations as well. This consistency is a highly desirable feature of sustainable CDC plan design.

Notably, all values of quantile-inequity remain below 2.5% for the CDC plan with active members, and under 1.6% for the decumulation-only CDC design. This indicates that, regardless of the specific values of the design parameters n and ξ , the disparity between the most and least favourable outcomes across cohorts is minimal. This observation is further reinforced by the fact that the median inequity remains below 10 basis points for the CDC plan with active members, and falls below 2 basis points for the decumulation-only design. These results demonstrate that, under typical scenarios, across the full range of design parameters considered, no cohort experiences materially different outcomes relative to others, regardless of whether the plan includes ongoing contributions from active members or operates solely on a decumulation basis. In other words, intergenerational risk-sharing mechanisms introduced by this generalized design framework for CDC plans lead to equitable and fair plan design overall.

Chapter 6

Concluding remarks

This report developed and examined a unified, flexible framework for the design of CDC pension plans. By introducing two core design parameters—the immediate experience-recognition rate and the smoothing-window length—and by considering both active-member and decumulation-only configurations, the framework encompassed the full spectrum of CDC structures proposed in the literature. Three adjustment mechanisms were analyzed in each setting: full and immediate recognition of mortality and investment experience, deferred recognition over a finite horizon, and a hybrid combination of the two.

Stochastic simulation results demonstrated the inherent trade-offs faced by plan sponsors and trustees. Plans that incorporate immediate experience recognition deliver stronger benefit security, as measured by realized IRRs and reduced serial disappointment, but at the expense of greater short-term volatility. Conversely, extensive smoothing markedly attenuates benefit fluctuations—reducing roughness and increasing IR smoothness—but can generate persistent intergenerational transfers if the smoothing horizon substantially exceeds members’ remaining lifetimes. Our hybrid approach enabled stakeholders to calibrate these dynamics along a spectrum, balancing security, stability and equity to suit specific demographic and regulatory needs and risk-sharing preferences.

The multidimensional evaluation framework introduced in Chapter 5 provided transparent and interpretable metrics for assessing plan performance. Benefit security measures, stability indicators, and intergenerational equity metrics together offered a comprehensive lens through which to compare alternative designs. Importantly, the results suggested that, with careful choice of recognition rate and smoothing window, it is possible to achieve substantial welfare gains without imposing systematic losses on any cohort—thereby reconciling key policy objectives.

This research project generated several avenues for future research. One promising direction involves refining the underlying assumptions of the framework, particularly concerning

mortality and investment return modelling. Incorporating mortality improvements over time and capturing systematic longevity risk would yield more realistic assessments of benefit dynamics and risk-sharing outcomes. On the financial side, extending the model to include stochastic interest rate processes, variable hurdle rates, and dynamic (rather than static) asset allocation strategies would enhance the robustness of simulated outcomes and allow for richer plan design experimentation. In addition, exploring behavioural dimensions—such as heterogeneity in member risk aversion, preferences over income stability, and contribution flexibility—would provide deeper insight into how individuals interact with collective pension arrangements. These enhancements would further strengthen the framework’s applicability in real-world settings and contribute to a more comprehensive understanding of accumulation–decumulation dynamics in CDC schemes.

In sum, this report offers a robust, generalized design toolkit for CDC pension plans, equipping policymakers, trustees, and plan sponsors with the means to tailor intergenerational risk sharing to the needs of their membership. By making explicit the trade-offs between security, stability, and fairness, it lays the groundwork for more resilient and equitable retirement income systems.

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Appendix A

Code

This GitHub repository consists of the following resources:

- Code for fitting model parameters to real-world data.
- Code for generating asset and mortality scenarios discussed in Chapter 2.
- Code to simulate plan dynamics under all approaches discussed in Chapter 3.
- Code for calculating performance metrics discussed in Chapter 5.
- Code for plotting figures shown in Chapters 4 and 5.
- Bash files for running simulations in parallel using SLURM clusters.
- README file explaining project and folder structure.